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ENGINEERING APPLICATIONS
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BY

V. KARAPETOFF

PART I.
PROBLEMS ON MACHINE DESIGN

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PREFACE.

This is not a book on calculus or analytic geometry (the market is flooded with them); nor is this a book on engineering or any branch of it. *The book is intended to enable an engineer to make a better and more extended use of higher mathematics in his work.* The purpose of the book may be best amplified by a parable.

In a manual-training school (on the moon) machinist apprentices were taught their trade in the following manner: During the first year they had a highly theoretical course on the subject of various tools used on lathes, planers, boring mills, milling machines, etc. The shapes of the tools were derived and explained in detail on complicated drawings; most general theorems were proved concerning these tools; it was shown how to design these tools, not only for a few simple practical cases, but principally for many hypothetical cases which were supposed to be of some importance on Mars. This latter part of the course was justified on the plea of mental gymnastics. No actual machine-tools were provided in this department and no practice was afforded the student in the use of the tools.

During the next two years the students were required to finish, fit, and assemble the parts of various engines and other pieces of mechanical apparatus. Had they been previously trained in the use of machine-tools, their shop-work would have been much simplified. But their highly theoretical information about tools was of no use to them, even if they had not forgotten it during the summer vacation. At any rate, they did not possess enough manual skill to put even an iron rod in a lathe and turn it down by a millimeter or two. So they preferred to use the old-fashioned chisel and file, and finished most of their parts in a vise. And their teachers acquiesced in this way of doing things, partly because they did not consider it to be their duty to teach the use of tools, since that was a part of the freshman-year instruction and was taught by another department, and partly because, having gone themselves through a similar school, they considered such a lack of correlation in teaching to be inevitable. Some teachers even considered this state of affairs to be a fundamental law of nature, second only to the law of gravitation.

But a heretic, an anarchist, an iconoclast, is sure to appear on the stage sooner or later. Such a one made his appearance one day and began undermining the pillars of the time-honored system. He claimed that it was wrong to instruct freshmen in the theory of the tools, without having applications and practical shop work going hand in hand. His idea was to show the student one or two simple tools and to apply them immediately to finishing a few pieces of work in a suitable machine-tool. In other words, according to this man, *knowledge and skill, or science and art, ought to be acquired simultaneously.* The reformer insisted that with this method of instruction the knowledge gained would become organic with the student, instead of being on the surface only. Besides, said he, modern psychology shows that interest is a paramount factor in education, and applications are always more interesting to an average student than a general theory. He was perfectly willing to grant that with his method, considerably less ground could be covered in pure theory during the first year; but then he claimed that machinist apprentices needed only a few simple tools during the next year's work, and that enough time could be spared later on to give an advanced course in the theory of special and fancy tools to those who desired it. This time could be spared, because the practical knowledge and skill in the use of tools, acquired during the first year, would enable the students to accomplish their shop-work in less time.

While it may seem incredible that mechanic arts should be taught anywhere in the ridiculous way described above, yet it is a fact that mathematics is most universally taught to engineering students in a similar fashion, without any correlation whatever with their profession. The student is first filled with analytics and calculus, as if he expected to become a pure mathematician. Then, very little of this mathematics is used in the engineering courses, partly because the students find it very difficult to follow, and partly because many professors of engineering themselves have not enough grounding in mathematics to feel at ease in it and to make it interesting and attractive to their students. The case is somewhat similar to that of modern languages. The engineering faculties insist that students shall acquire proficiency in French and German, while the students know perfectly well that most of their teachers never read foreign books or magazines. The old saying about the mote and the beam in the eye involuntarily suggests itself.

The heretic who first raised his voice against the unsatisfactory teaching of mathematics to engineering students, and who showed the way out of it, was the noted British engineer and educator, John Perry. In his "Calculus for Engineers" (1897), he limits the theory to a minimum, and gives a large number of examples taken from various branches of engineering. After the student has mastered the differentiation and the integration of a few principal functions and has solved a large number of practical engineering problems, involving these functions, he is led into a more general theory of differentiation and integration, somewhat as it is done in the ordinary courses in calculus.*

Perry's pioneer work, and the so-called "Perry Movement" for teaching mathematics in a practical way, will forever remain important monuments in the history of engineering education. Like any other pioneer work, Perry's Calculus proved to possess some defects when used as a regular textbook, and several similar works by other writers followed it (see Appendix).

This book differs from that of Perry and his followers in two respects:

(1) An elementary knowledge of analytics and calculus is presupposed;

(2) The arrangement of the chapters is according to the engineering topics and not according to the mathematical functions or operations.

It is not necessary to burden a book of this character with the theory of analytic geometry and calculus, because it is treated in numerous excellent works, large and small. The names of some of these books are given in the list at the end of this book. This book is intended to supply the engineering applications and reasoning not found in the other books. The arrangement by engineering topics permits the giving of more interesting examples than if they were arranged according to the mathematical functions or operations; namely, having explained to the student the purpose of a simple engineering appliance, such as a belt or a flywheel, the teacher can give a number of mathematical examples on these appliances, from the most elementary to the most difficult, without going into engineering practice. In other words, the aim is to enable a teacher of mathematics to give a large number of practical problems, with very little engineering information. All the necessary explanation in regard to the construction and use of each device is given at the beginning of each chapter, so that it is not necessary to look up any books on machine design. However, a list of such engineering works will be found at the end of this book.

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* For a detailed development of this idea in application to all engineering subjects, see V. Karapetoff, "On the Concentric Method of Teaching Electrical Engineering," in *Trans. Amer. Inst. Electr. Engrs.*, Vol. 26 (1907), p. 1441; also his paper "On the Concentric Method of Education in Engineering," *Proc. Soc. Promotion Eng. Educ.*, Vol. 16 (1908), p. 258.

In accordance with the above-described purpose and plan of the book, the following possible uses of it are suggested:

(*a*) As a problem book in connection with a regular course in analytics and calculus.

(*b*) As a textbook in a supplementary course (after the completion of a brief course in calculus) taught in the department of mathematics for the purpose of fixing the mathematical operations in the mind of the student and preparing him for the engineering subjects.

(*c*) As a textbook in a course in Engineering Mathematics, taught in the junior or in the senior year by an engineer, to illustrate the methods of engineering research and analysis.

(*d*) As a text- or reference-book in a seminar for graduate students in engineering. Most of these men come to colleges with a knowledge of calculus which is anything but good, and consequently are handicapped in their research and in understanding the literature of the subject upon which they are working.

(*e*) As a study book for teachers in engineering and for practicing engineers, who require mathematics in their work and feel that they need a "brushing up" in order to be able to follow intelligently books, magazines, and transactions of engineering societies.

The author wishes to emphasize that while he has selected his problems from the theory of machine elements, the book is equally useful to engineers of all kinds. It is the reasoning and the practice in the use of calculus and analytics in the solution of engineering problems that is of importance, and not the particular kind of appliances to which the problems refer. However, the author has also collected for his students a large number of problems taken from Hydraulics, Strength of Materials, Thermodynamics, and Electrical Engineering. It is his intention to publish these problems in similar small volumes.

The author wishes to acknowledge with thanks the moral support, encouragement, and assistance on the part of some members of the mathematical faculty of Cornell University—in particular, Professors James McMahon and F. R. Sharpe. Mr. John G. Pertsch, Jr., instructor in electrical engineering in Sibley College, read the manuscript and the proofs of the book and checked the solutions of the problems. To him the author is under obligation for the care with which the work was done.

CORNELL UNIVERSITY, ITHACA, N. Y.
September, 1911.

A DIALOGUE

Between the Prospective User of the Book and the Author.

Prospective User. — You assume in your book that I can differentiate and integrate almost any function, but such is not the case.

Author. — I assume only that sometime you have studied the elements of calculus, and that by referring to your old books you can refresh your memory as to the processes of differentiation and integration. The specific information, such as formulæ, routine transformations, etc., you will find in the reference books by Seaver, Claudel, and Peirce, mentioned in the Appendix. You should have at least Seaver's book for reference, while studying my book; this will save you considerable time.

P. U. — Yes, but even where you indicate the solution you seem to omit important steps in transformations; at least so it appears to me because your deductions and proofs are so short.

A. — Engineering problems and deductions must be short, because the assumptions made in the beginning are somewhat crude anyway. In solutions I omit only such steps as are taken according to the well-known rules of elementary or higher mathematics. The student has no choice there and is sure to fill in the omitted parts correctly. If I should give the details of tranformations, the principal reasoning would be obscured.

P. U. — This may be so, but I had my algebra and trigonometry so long ago that I am not certain even about the simplest transformations and relations, and I should much rather have them indicated in your book.

A. — No one remembers all the formulæ, relations, and transformations, not even a professional mathematician. I want you to go back to your old books and to the reference books of which I spoke before, and get the specific information which you need in each problem. In this way you will learn much more and will remember better than if I gave you all the steps. The profit derived is in proportion to the individual effort made and not according to the number of pages gone over in a perfunctory way.

P. U. — I am willing to grant you this point, but I want you to understand that I am no exception as far as the knowledge of elementary mathematics is concerned. I think my knowledge of it was above that of an average student, but I never used enough of algebra or trigonometry to make my knowledge organic. Do you not think that we ought to have, in the freshman year, problems on engineering applications of elementary mathematics, before we embark on analytic geometry at all? My principal difficulty with higher mathematics was that I could not follow the algebraic transformations fast enough, and therefore lost the principal thought. I always felt like a man who came to the theater in the middle of the second act, and tried to follow the plot of a complicated play.

A. — I fully agree with you there. Colleges of engineering ought to see to it that the course in mathematics begins with a series of practical problems in elementary mathematics, before analytics or calculus are touched at all. Saxelby's book, mentioned in the Appendix, is very good for the purpose. I trust that someone will write a small book on the engineering applications of elementary mathematics. As for you, I am certain that you will incidentally acquire a considerable fluency in algebra and trigonometry by solving the problems in my book.

P. U. — You do not give much numerical work in your book. Do you not think that fluency in computations is quite essential for the engineer?

A. — I do, and I require numerical computations in those problems where such computations or the numerical results bring out a new point. To do more than this would distract attention from the principal aim of the book. This aim, as you understand, is to train the engineering student in such reasoning in his profession where higher mathematics can be used to advantage. I even avoided giving numerical coefficients wherever possible, because such are given in engineering pocket books and handbooks. Please do not ascribe much importance to the numerical data in the problems. They are given merely for exercise.

P. U. — You have just mentioned engineering reasoning. Is it any different from that used in other sciences?

A. — I should have said reasoning in application to engineering problems. It means first to "size up" the problem in regard to the physical laws involved and the empirical assumptions to be made. After this, the problem is to be clothed in a mathematical language. Then comes the solution, with the necessary transformations, approximations, etc. The result must be analyzed as to its applicability to the particular practical problem. This means going back to the physical laws and to the engineering assumptions made. It is always advisable to apply the solution to at least one set of numerical data, or, still better, to some average data and to extreme data. The numerical results often suggest a new solution, different assumptions, another approximation, or at least a different form of the solution.

P. U. — What you have just enumerated really requires a man to be a competent engineer, and at the same time a first-class mathematician. This is a rare combination. Cannot two men, an engineer and a mathematician, solve such problems together, in a coöperative way?

A. — Yes, they could if they understood each other better than they do now. They could if the engineer were able to put his problem into the mathematical language, stripping it of all the descriptive factors which cannot be expressed by formulæ; and if the mathematician had a little engineering feeling which would enable him to suggest more suitable fundamental assumptions, and to present the solution in a form which would not scare a practician by its very size. My book is intended to bring these two classes of men into a closer relation. Suppose that you were in France and had to write a letter in French. If you knew the French language well enough to explain the contents of the proposed letter to someone, you would find any number of French people who would write this letter for you in correct French. Or else, you would have to find a French person who knew enough English to understand your purpose.

P. U. — Don't you think that this lack of understanding, this chasm between theory and practice in this country, is gradually closing up?

A. — I most certainly do, and I will tell you the reason why. At first we had in this country only practical practicians and theoretical theoreticians. They were like two persons who spoke different languages and did not care for each other's point of view. Now we have two classes of middlemen in the field: theoretical practicians and practical theoreticians. This applies not only to engineering and mathematics, but to nearly all forms of human activity. A practical theoretician is usually a professor of applied mathematics or physics, who fully understands the language of the theoretical mathematician and at the same time is full of desire of applying the results of the theory to the solution of practical problems, though he may not know exactly what is needed in practice. A theoretical practician is usually a professor of engineering, or an expert in the employ of some industrial corporation. He is fully conversant with the practical conditions, and sees the need of solving certain practical problems in a rational way by means of mathematics, but he is not enough of a mathematician to tackle these problems himself. Now, the theoretical practician and the practical theoretician have enough in common to understand each other, to sympathize with each other, and to solve problems coöperatively. In so doing they draw for information upon the theoretical theoretician and the practical practician, and thus bring these two classes of people to serve each other.

P. U. — Then, using your analogy, I would say that the theoretical theoretician is a man who speaks French only, and a practical practician is a man who speaks English only. A practical theoretician is a Frenchman who is fond of English and dabbles a little in it. A theoretical practician is an Englishman who loves French and understands it a little when it is not spoken too fast. These two men become friends, and through them the "French-only" person is made to understand the "nothing-but-English" individual.

A. — This is exactly what I meant.

P. U. — Well, now that I understand the purpose of your book and the method of studying it, the task does not seem so difficult to me. I guess I'll get busy at once on that first chapter about the inclined plane.

A. — Yes, and see to it that you climb up that plane, and not roll down.

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ENGINEERING MATHEMATICS.

CHAPTER I.

INCLINED PLANE AND SCREW.

A LOAD L, weighing P kg. (Fig. 1), is hauled up an inclined plane AB at a uniform speed, by a horizontal force of Q kg. It is required to find θ, the angle of inclination of the plane, such that the mechanical efficiency of the arrangement may be a maximum. The mechanical efficiency is defined as the ratio of the useful work performed to the total energy expended.

Let the load move a distance s along the inclined plane; the useful work performed against the force of gravity P is then $P \cdot s \sin \theta$, where $s \sin \theta$ is the distance traversed by the load L against the direction of the resisting force P. Similarly, the work expended by the moving force Q is $Q \cdot s \cos \theta$. According to the definition given above, the efficiency

$$\eta = \frac{Ps \sin \theta}{Qs \cos \theta} = \frac{P}{Q} \tan \theta. \quad . \quad . \quad . \quad . \quad (1)$$

The amount of work expended by the moving force Q is larger than that merely required to lift the weight, because the force Q has, in addition, to overcome the friction between the load and the plane. In order to determine the ratio between P and Q, it is therefore necessary to consider the force of friction. This force is found, by experiment, to be proportional to the normal pressure between the rubbing surfaces, and to be directed so as to oppose the motion. In Fig. 1, N represents the normal pressure of the plane upon the load, and μN is the corresponding force of friction. The factor of proportionality μ is called the coefficient of friction; its numerical value depends upon the materials, finish, and lubrication of the rubbing surfaces.

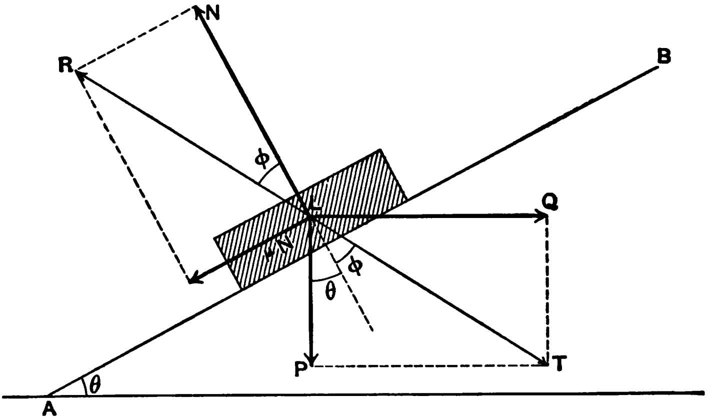

Fig. 1. — Diagram of forces for an inclined plane; the load being moved upward by a horizontal force.

The force R, being the resultant of N and μN, represents the total reaction of the inclined plane upon the load. It will be seen that this resultant is inclined at an angle ϕ from the normal, such that

$$\tan \phi = \frac{\mu N}{N} = \mu. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

This angle is called the angle of friction; the amount of friction between two surfaces can be indicated by giving either the value of μ or the angle ϕ, the two being connected by Eq. (2). The load P is moving, by assumption, at a uniform rate under the influence of the forces P, Q, and R; hence these three forces must be in equilibrium. In other words, the resultant, LT, of P and Q must be equal and opposite to the force R. It will be seen from the figure that the angle $PLT = \theta + \phi$, so that

$$Q = P \tan (\theta + \phi). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Substituting this value of Q into Eq. (1) gives

$$\eta = \frac{\tan \theta}{\tan (\theta + \phi)}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Thus, the efficiency depends only upon the angle of inclination θ and upon the angle of friction ϕ, and is independent of the weight of the load. To find the value of θ, for which the efficiency becomes a maximum, we equate to zero the first derivative of η with respect to θ. This gives the following equation for θ:

$$\frac{1}{\cos^2 \theta} \cdot \tan (\theta + \phi) = \frac{1}{\cos^2 (\theta + \phi)} \cdot \tan \theta.$$

The efficiency η is a maximum when θ satisfies this equation. Replacing the tangents by the ratios of the corresponding sines and cosines, it is found, after reduction, that

$$\sin 2 (\theta + \phi) = \sin 2 \theta.$$

Now the sines of two angles are equal, either when the angles themselves are equal, or when the sum of the angles is 180 degrees. The first assumption leads to $\phi = 0$, and thus $\mu = 0$; in other words, there is no friction. In this case the efficiency is equal to 100 per cent at any value of θ, since there is no source of loss of work. Moreover, in this case η in Eq. (4) is a constant $= 1$, so that there is no question of a maximum. The second assumption gives

$$2 (\theta_1 + \phi) + 2 \theta_1 = 180 \text{ degrees},$$

from which

$$\theta_1 = 45^\circ - \frac{\phi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Here θ_1 is the value of θ for which η is a maximum.

The smaller the friction the nearer θ_1 approaches the value of 45 degrees; the presence of friction reduces the value of this angle of inclination. Thus, for instance, if the angle of friction $\phi = 10$ degrees, the best angle to use for an inclined plane is 40 degrees; if $\phi = 20$ degrees, $\theta_1 = 35$ degrees, etc.*

Prob. 1. — The coefficient of friction between cast iron and steel, under certain conditions, is 0.347. At what angle θ_1 is the efficiency of the inclined plane a maximum? *Ans.* 35° 26′.

Prob. 2. — Show that the Eq. (5) is equivalent to the relation $\tan 2\,\theta_1 = \frac{1}{\mu}$; check the answer of Prob. 1, using this formula.

Prob. 3. — Show that $\eta_{\max} = \tan^2 \theta_1$, by substituting Eq. (5) into formula (4).

Prob. 4. — Plot curves of the most advantageous angles of inclination, and of the corresponding efficiencies, to values of coefficient of friction as abscissæ, between the limits of $\mu = 0$, and $\mu = 0.5$.

Ans. The extreme ordinates of the curves are:

μ	θ_1	$\eta_{\max}$ (in per cent)
0	45°	100
0.5	31° 43′	38.1

Prob. 5. — Prove from Eq. (4) that the efficiency is less than 50 per cent when $\theta = \phi$.

Prob. 6. — Suggest a simple graphical method for determining the value of η from Eq. (4). *Hint:* — In Fig. 2, select $ac = 100$ per cent; then $ab = \eta$, in per cent.

Prob. 7. — Show that Eq. (4) can be represented in the form $\eta = \tan(\theta_1 + \beta)\tan(\theta_1 - \beta)$, where θ_1 is the most advantageous angle defined by Eq. (5), and β is the deviation of the actual angle θ from θ_1; in other words, $\theta = \theta_1 + \beta$. *Hint:* $\tan(\theta + \phi) = \tan(\theta_1 + \beta + 90° - 2\,\theta_1) = \cot(\theta_1 - \beta)$.

Prob. 8. — The load slides down the inclined plane, instead of being hauled up; the force Q resists the motion so as to maintain a uniform speed. The force of friction μN (Fig. 1) is now directed upward, instead of downward, and the angle $PLT = \theta - \phi$. The force P is in this case the moving force, and the efficiency

---

* The same rule applies to the square-threaded screw, which can be considered as an inclined plane cut on a cylinder. However, in designing screws the efficiency is often sacrificed for other practical considerations, and the thread angle is usually selected considerably below the value corresponding to Eq. (5).

$$\eta' = \frac{Qs\cos\theta}{Ps\sin\theta} = \frac{Q}{P}\cdot\frac{1}{\tan\theta} = \frac{\tan(\theta - \phi)}{\tan\theta} \quad . \quad . \quad . \quad . \qquad (4a)$$

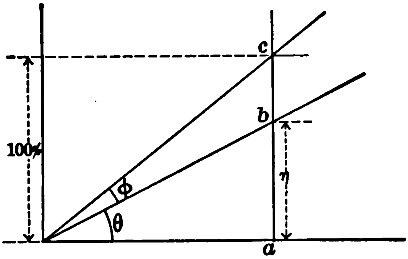

Fig. 2. — Graphical determination of efficiency, η, according to Eq. (4).

Find the maximum value of this efficiency at a given angle of friction and with various inclinations of the plane to the horizontal.

Ans. $\eta'_{\max} = \cot^2\theta_2$, where $\theta_2 = 45^\circ + \frac{\phi}{2}$.

Prob. 9. — A load is moving up and down an inclined plane, or is being raised and lowered by means of a power screw. Show that the efficiency is higher when the load is going up, provided that the angle θ is less than 45 degrees.

Solution. — It is required to prove that

$$\frac{\tan\theta}{\tan(\theta + \phi)} > \frac{\tan(\theta - \phi)}{\tan\theta},$$

or that $$\tan^2\theta > \tan(\theta + \phi)\tan(\theta - \phi),$$

when $\tan\theta$ is < 1.

We have

$$\tan(\theta + \phi)\tan(\theta - \phi) = \frac{\tan^2\theta - \tan^2\phi}{1 - \tan^2\theta\tan^2\phi}$$

$$= \frac{1 - \tan^2\phi/\tan^2\theta}{1 - \tan^2\phi\tan^2\theta}\cdot\tan^2\theta.$$

Since $\tan\theta$ is < 1, the quantity subtracted from the unity in the numerator of the fraction is larger than the quantity subtracted from the unity in the denominator. Hence the fraction is proper, and the inequality holds true. In a similar manner, when θ is above 45 degrees, $\tan\theta$ is > 1, and the fraction is improper. The sign of the inequality must then be reversed; this means that in this case the efficiency is higher when the load is descending.

Prob. 10. — A load is being hauled up an inclined plane by a force parallel to the plane (Fig. 3). Find the inclination of the plane at which the efficiency reaches its maximum.

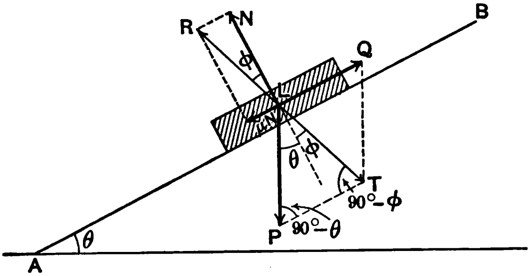

Fig. 3. — Diagram of forces for an inclined plane; the load being moved upward by a force parallel to the plane.

Ans. The efficiency reaches its maximum value of 100 per cent when the plane is vertical; in this case there is practically no pressure and consequently no friction between the load and the plane.

Solution. — When the load has traveled a distance s along the plane, the work done against the force of gravity is $P \cdot s \sin\theta$, while the work expended by the moving force Q is Qs. Therefore the efficiency

$$\eta = \frac{Ps \sin\theta}{Qs} = \frac{P \sin\theta}{Q}.$$

But from the triangle LPT

$$\frac{P}{Q} = \frac{\sin(90° - \phi)}{\sin(\theta + \phi)},$$

so that

$$\eta = \frac{\cos\phi \sin\theta}{\sin(\theta+\phi)} = \frac{1}{1 + \tan\phi \cot\theta} \quad . \quad . \quad . \quad . \quad (6)$$

The efficiency is the nearer 100 per cent, the smaller the product of $\tan\phi \cdot \cot\theta$. This product is equal to zero when either $\phi = 0$ or $\theta = 90°$. The first case corresponds to an ideal plane without friction; the second assumption gives a vertical plane.

Prob. 11. — Referring to the preceding problem, let the load move down the plane. By analogy with Prob. 8, the efficiency is

$$\eta' = \frac{\sin(\theta - \phi)}{\cos\phi \sin\theta} = 1 - \tan\phi \cot\theta \quad . \quad . \quad . \quad . \quad (6a)$$

Prove that $\eta > \eta'$, that is to say, the efficiency is higher in ascending the plane than it is in descending. *Hint:* $1 > 1 - N^2$; consequently $\frac{1}{(1+N)} > 1 - N$. Another way to prove the proposition is to expand η into an infinite series by dividing 1 by $(1+N)$. It must be remembered that the expression for η' presupposes that

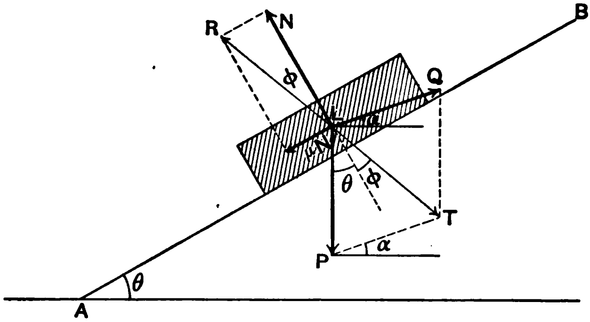

Fig. 4. — Diagram of forces for an inclined plane; the load being moved upward by a force inclined at an angle α from the horizontal.

$\tan\phi \cdot \cot\theta < 1$; otherwise the definition of efficiency given above would not hold without an additional qualification.

Prob. 12. — A load moves along an inclined plane at a uniform speed; the force Q (Fig. 4) acts at an angle α from the horizontal. Find an expression for the efficiency of the mechanism, when the load moves up the plane; and also for the case, when the load is moving down the plane.

Solution. — When the load moves up the plane (Fig. 4), P resists the motion and Q is the moving force. The efficiency is then, by definition, the ratio of the useful work performed against the force P to the work actually expended by the moving force Q; or

$$\eta = \frac{Ps \cdot \cos(P,s)}{Qs \cdot \cos(Q,s)} = \frac{P \sin\theta}{Q \cos(\theta - \alpha)}.$$

But from the triangle of forces LPT

$$\frac{P}{Q}=\frac{\sin LTP}{\sin PLT}=\frac{\cos(\theta+\phi-\alpha)}{\sin(\theta+\phi)},$$

so that

$$\eta=\frac{\cos(\theta+\phi-\alpha)\sin\theta}{\sin(\theta+\phi)\cos(\theta-\alpha)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

When the force Q is horizontal, $\alpha=0$, and the foregoing expression for efficiency becomes identical with Eq. (4). When Q is parallel to the plane, $\alpha=\theta$, and Eq. (7) checks with Eq.(6).

When the load slides down the plane, by analogy with Prob. 8 we get for the efficiency

$$\eta'=\frac{\sin(\theta-\phi)\cos(\theta-\alpha)}{\cos(\theta-\phi-\alpha)\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

This expression reduces to Eq. (4a) when $\alpha=0$, and to Eq. (6a) when $\alpha=\theta$.

Prob. 13. — For the values of θ_1 and ϕ found in Prob. 1, plot an efficiency curve, when the angle α, at which the force Q is acting, varies from + 90 degrees to a negative value at which the efficiency becomes zero.

Ans.				
$\alpha=$	$+90°$	$+40°$	$0°$	$-35°\,26'$
$\eta=$	100	69.1	50.6	0

Prob. 14. — Determine the value of the angle of inclination θ in Eq. (7), at which the efficiency becomes a maximum, with given values of ϕ and α.

Ans. $\theta_1=45°+\frac{\alpha}{2}-\frac{\phi}{2}$.

Hint.— Use the transformation $2\cos A\sin B=\sin(A+B)-\sin(A-B)$, before differentiating Eq. (7).

Prob. 15. — Using the answer of the preceding problem, prove that

$$\eta_{\max}=\frac{\sin^2\theta_1}{\cos^2(\theta_1-\alpha)},$$

and show that the result checks with that obtained in Prob. 3, when $\alpha=0$.

Prob. 16. — Assuming $\mu=0.25$, plot a curve of values of θ_1, the most advantageous angle of inclination of the plane, for various angles α of hauling; also a curve of the corresponding values of efficiency.

Ans.				
$\alpha =$	$-76°$	$0°$	$+76°$	$+90°$
$\theta =$	0	38	76	83
$\eta_{\text{max.}} =$	0	61	94.1	100

Prob. 17. — At what angle α must the force Q be applied in order to obtain a maximum efficiency on a given inclined plane?

Ans. The efficiency is 100 per cent when the load is simply lifted vertically; when Q acts at an angle to the vertical, the efficiency is the higher, the nearer the direction of Q approaches to the vertical.

Solution. — The efficiency is a maximum when the term

$$\frac{\cos(\theta + \phi - \alpha)}{\cos(\theta - \alpha)} \text{ in Eq. (6) becomes a maximum.}$$

But
$$\frac{\cos(\theta + \phi - \alpha)}{\cos(\theta - \alpha)} = \cos\phi - \sin\phi\tan(\theta - \alpha),$$

so that η is a maximum when $\tan(\theta - \alpha)$ has the largest possible negative value consistent with the problem. This corresponds to $\alpha = 90°$.

Prob. 18. — The preceding theory applies essentially to a square-threaded screw, which can be considered as an inclined plane cut on a cylinder. However, in calculating the efficiency of a screw, it is necessary in some cases to take into account the friction in the pivot or collar, in addition to the friction between the screw proper and the nut. When the driving force Q is horizontal, this additional collar friction is proportional to the weight lifted, and can be represented by a horizontal force fP, where f is a constant depending upon the coefficient of pivot friction and upon the dimensions of the pivot. The problem is to find an expression for efficiency, analogous to Eq. (4), when the screw is lifting a weight P.

Ans. Eq. (3) becomes: $Q = P\tan(\theta + \phi) + fP$, so that

$$\eta = \frac{\tan\theta}{\tan(\theta + \phi) + f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Prob. 19. — Find the value of the thread angle θ which converts the expression (8) into a maximum.

Ans. $\tan\theta_1 = -k + \sqrt{k^2 + \frac{k}{\mu}}$, where $\mu = \tan\phi$ and $k = \frac{(\mu + f)}{(1 - \mu f)}$.

Hint.— Denote $\tan \theta = z$, so that

$$\eta = \frac{z}{\dfrac{z+\mu}{1-z\mu}+f};$$

introduce k into this expression and differentiate with respect to z.

Prob. 20. — The coefficient of friction μ between the screw and the nut is estimated to be 0.18 for a certain kind of service. Tabulate values of the best thread angles and of the corresponding efficiencies, for values of f from 0 to 0.20.

Ans. $f = 0$ $\theta_1 = 39° 54'$ $\eta = 69.9$
0.20 48° 40' 61.3

Prob. 21. — When the movement of the screw is downward we find, by analogy with Probs. 8 and 18,

$$\eta' = \frac{\tan(\theta - \phi) - f}{\tan \theta}.$$

Find the value of θ at which the efficiency reaches its maximum.

Ans. $\tan \theta_2 = k + \sqrt{k^2 + \frac{k}{\mu}}$ (see Prob. 19).

CHAPTER II.

FRICTION IN JOURNALS.

A HORIZONTAL shaft (Fig. 5) revolves in a bearing that fits it well. The problem is to determine the force of friction between the bearing and the journal (the part of a shaft inside of a bearing is usually called the journal). This force of friction, which is tangential to the surface of the journal and opposes the motion of the shaft, is usually assumed to be proportional to the normal pressure between the bodies in contact. In the case under consideration, this pressure may be different in different places on the journal, so that the total force of friction is obtained by integrating the infinitesimal forces of friction over the total surface of the bearing.

Let the normal pressure per square centimeter of the surface of the bearing be denoted by p. This unit pressure, generally speaking, is different for different horizontal strips of the bearing, the weight of the shaft being distributed non-uniformly. In mathematical language, p may be said to be a function of the angle θ (Fig. 5). The normal force acting upon an infinitesimal strip, $l\,rd\theta$, of the bearing is $p\,l\,rd\theta$; the corresponding force of friction is $\mu\,p\,l\,rd\theta$, where μ is the coefficient of friction. The numerical value of μ varies within wide limits according to the materials used, and to the kind of lubrication.

Integrating, gives the total force of friction,

$$F = 2\int_0^{\frac{\pi}{2}} \mu plr\,d\theta = A\int_0^{\frac{\pi}{2}} \mu p\,d\theta. \quad . \quad . \quad . \qquad (1)$$

Here $A = 2rl$ is the so-called projected area of the bearing, or the cross section of the journal by the horizontal plane passing through its axis. The use of this projected area is convenient in practical calculations pertaining to journals and bearings. The limits of integration should be $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$, but since the bearing is symmetrical it is sufficient to calculate the force of friction for one quadrant and multiply the result by 2.

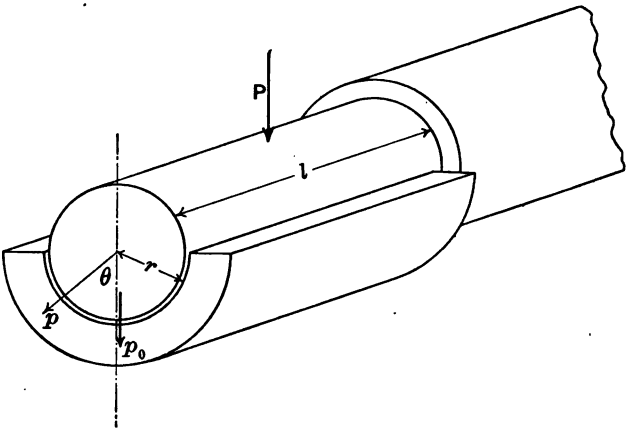

Fig. 5. — The journal of a shaft resting in a bearing.

Assuming the coefficient of friction to be independent of the pressure p, Eq. (1) is simplified to

$$F = A\mu \int_0^{\frac{\pi}{2}} p\, d\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In order to perform the integration, p must be given as a function of the angle θ. The actual distribution of the pressure over the surface of the bearing is not definitely known. Various reasonable assumptions in regard to this distribution are made in the problems that follow.

But whatever the distribution of pressures, the sum total of the vertical projections of these pressures must be equal to the load P resting upon the bearing, in order that the bearing may be in equilibrium. This gives a general condition which the unit pressures must satisfy in all cases; namely:

$$P=2\int_0^{\frac{\pi}{2}} plr\,d\theta\cos\theta=A\int_0^{\frac{\pi}{2}} p\cos\theta\,d\theta. \quad . \quad . \quad (3)$$

All the problems that follow are solved by the use of Eq. (2) and (3), when the coefficient of friction is assumed constant, or by Eq. (1) and (3), when this coefficient is variable.

Prob. 1. — In a new bearing with a perfect fit the normal pressure p can be assumed the same over all the surface of the journal; in other words,

$$p = p_0 = \text{Constant}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Express the force of friction F as a function of the load P and the coefficient of friction, μ.

Solution. — Eq. (2) gives

$$F = A\mu p_0\cdot\frac{\pi}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and from Eq. (3)

$$P = Ap_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Combining these two equations in order to eliminate the unknown pressure p_0, we obtain

$$F=\mu P\cdot\frac{\pi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From this formula the resisting force of friction caused by a bearing can be calculated, when the load P resting upon the bearing, and the coefficient of friction, μ, are known.

Prob. 2. — Bearings are always made of a softer metal than shafts, so that, in proportion as the bearing wears out, more and more pressure is exerted on the bottom of it, and less on the sides. After the bearing has been in use for some time, it is ground by the journal into such a shape that further wear does not alter its shape, but merely lowers the position of the shaft. Let S in Fig. 6 be this final shape of the surface of the journal. The surface S' to which the bearing is worn after some further use is identical with S, the vertical wear w being the same at all points. The problem is to find the distribution of the normal pressures p such that gives a uniform vertical wear.

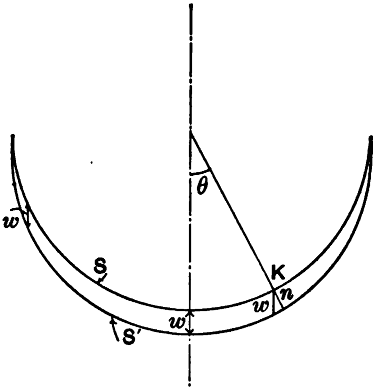

Fig. 6. — Two consecutive surfaces of a bearing, with a constant vertical wear.

Solution. — Let the normal wear at a point K, defined by the central angle θ, be n. This wear can be assumed proportional to the normal pressure exerted at this point;* in other words, $n = Gp$. Here G is a constant which depends upon the material of the bearing, the lubrication, and the speed of the shaft. On the other hand, we have from the figure: $n = w \cos \theta$, so that $w \cos \theta = Gp$. Since, by assumption, both G and w are constant, we have

$$p = p_0 \cos \theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $p_0 = \frac{w}{g}$ is a constant. This is the required law of distribution of the normal pressure in a worn bearing. When $\theta = 0$, $p = p_0$, so that the constant p_0 represents the unit pressure at the bottom of the bearing. The pressure p decreases from the bottom of the bearing upward as the cosine of the angle θ. No normal pressure is exerted at the sides, where $\theta = 90°$.

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* Normal wear is proportional to the work of friction per unit area, so that at a given speed, and with a constant coefficient of friction, it is proportional to the normal pressure.

Prob. 3. — Find the relation between the load and the force of friction in a worn bearing, using Eq. (8).

Solution. Substituting the value of p into Eq. (2) and (3) and integrating, we find

$$F = A\mu p_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$P = Ap_0 \cdot \frac{\pi}{4} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Eliminating Ap_0 gives

$$F = \mu P \cdot \frac{4}{\pi} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Prob. 4. — A journal 15 cm. in diameter and 32 cm. long runs at a speed of 200 rev. per min. The weight supported by the bearing is 12 tons (one metric ton = 1000 kg.). Assuming the bearing to be worn, calculate: (*a*) the power lost in friction, with a coefficient of friction = 0.0032; (*b*) the average pressure upon the projected area; (*c*) the maximum unit pressure at the bottom of the bearing.

Solution. — (*a*) Using formula (11) we get: $F = 0.0032 \times 12{,}000 \times \frac{4}{\pi} = 48.89$ kg. The path traversed by the force of friction per second is $\frac{0.15\,\pi \times 200}{60} = 1.571$ m., so that the work lost is $48.89 \times 1.571 = 76.8$ kg-m. per sec. = 753 watts (one kilogram-meter per second is equivalent to 9.81 watts). (*b*) $\frac{P}{A} = \frac{12{,}000}{(15 \times 32)} = 25$ kg. per sq. cm. (*c*) From Eq. (10), $p_0 = 25 \times \frac{4}{\pi} = 31.8$ kg. per sq. cm.

Prob. 5. — Solve Prob. 4 under the supposition that the bearing is new. *Ans.* (*a*) 930; (*b*) 25; (*c*) 25.

Prob. 6. — Solve Prob. 3 with the assumption that the pressure varies according to the law $p = p_0 \cos^2 \theta$.

Ans. $F = \mu P \cdot \frac{3\pi}{8}$; $P = \frac{2}{3} Ap_0$.

Prob. 7. — Calculate friction in a bearing in which normal pressure varies according to the straight-line law $p = p_0 (1 - k\theta)$, where k is an empirical constant.

Ans. $P = Ap_0\left(1 - k\frac{\pi}{2} + k\right)$;

$$F = \mu P \left[\frac{\pi}{2} - k\frac{\pi^2}{8}\right] \div \left[1 - k\left(\frac{\pi}{2} - 1\right)\right].$$

Prob. 8. — The force of friction in a bearing was found from experiments to be expressed by the relation $F = 1.35\,\mu P$. Assuming the pressure to vary according to the straight-line law, as in Prob. 7, determine the value of the constant k, and the ratio of the smallest to the greatest unit pressure.

Ans. $k = 0.478$; $\frac{p_{\min}}{p_0} = 1 - k\frac{\pi}{2} =$ about 0.25.

Prob. 9. — Solve Prob. 7, assuming that the pressure varies as the square of the angle, that is to say, according to the formula $p = p_0(1 - c\theta^2)$, where c is an empirical constant.

Ans. $$P = Ap_0\left[1 - c\left(\frac{\pi^2}{4} - 2\right)\right];$$

$$F = \mu P\left[\frac{\pi}{2} - c\frac{\pi^3}{24}\right] \div \left[1 - c\left(\frac{\pi^2}{4} - 2\right)\right].$$

Prob. 10. — A bearing having a projected area of 300 sq. cm. supports a weight of 6 tons. Calculate the maximum pressure at the bottom of the bearing under the following assumptions: (*a*) The bearing is new; (*b*) the bearing is perfectly worn; (*c*) the pressure varies as $\cos^2\theta$; (*d*) the pressure varies according to a straight-line law, the pressure being zero at $\theta = 90°$; (*e*) same as (*d*), only the pressure at $\theta = 90°$, is equal to one-half of the maximum pressure; (*f*) the pressure varies according to a parabolic law, as in Prob. 9, the pressure being zero at $\theta = 90°$; (*g*) same as (*f*), only the pressure at $\theta = 90°$ is equal to one-half of the maximum pressure.

Ans. (*a*) 20; (*b*) 25.45; (*c*) 30; (*d*) 31.42; (*e*) 24.45; (*f*) 24.67; (*g*) 22.1 (in kg. per sq. cm.).

Prob. 11. — Using the data of the preceding problem, plot curves of the distribution of pressure against angles θ as abscissæ. Draw all the seven curves on the same sheet and to the same scale, to enable a direct comparison.

Ans. At $\theta = 45°$, the ordinates are: (*a*) 20; (*b*) 18; (*c*) 15; (*d*) 15.71; (*e*) 18.34; (*f*) 18.50; (*g*) 19.34 (in kg. per sq. cm.).

Prob. 12. — Show that the curve $p = p_0(1 - c\theta^2)$, used in Prob. 9, 10, and 11, is a parabola; reduce its equation to the standard form $y^2 = 2\,mx$. *Ans.* $\theta^2 = 2 \cdot \frac{1}{2\,p_0c} \cdot (p_0 - p)$.

Prob. 13. — In all the foregoing problems the coefficient of friction is assumed to be independent of the pressure. Some experiments indicate that within certain limits the coefficient of friction decreases with increasing friction. Solve Prob. 3 under the assumption that the coefficient of friction varies according to the law $\mu = \mu_0 - \mu' p$, where μ_0 and μ' are empirical constants.

Ans. $F = Ap_0\left(\mu_0 - p_0\mu'\frac{\pi}{4}\right)$, where p_0 is determined by Eq. (10).

Hint.—Use Eq. (1) instead of (2).

Prob. 14.—Solve Prob. 13 under the supposition that the coefficient of friction μ is directly proportional to the square root of the peripheral speed of the journal and inversely proportional to the square root of the unit pressure.

Ans. $\mu = \mu_1\sqrt{\frac{v}{p}}$, where μ_1 is a constant;

$$F = A\mu_1 \cdot \sqrt{vp_0}\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta} \cdot d\theta = 1.19\, A\mu_1\sqrt{vp_0}.$$

The maximum pressure p_0 is determined from Eq. (10).

Note.—The expression $\sqrt{\cos\theta} \cdot d\theta$ cannot be integrated in the finite form, and has to be calculated approximately, either by the trapezoid formula, or by the Simpson rule. According to the former, we have, taking ordinates every 5 degrees:

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta} \cdot d\theta = 5° \times \frac{\pi}{180°}\left[\frac{1}{2}\sqrt{\cos 0°} + \sqrt{\cos 5°} + \sqrt{\cos 10°} + \cdot\cdot\cdot + \sqrt{\cos 85°} + \frac{1}{2}\sqrt{\cos 90°}\right] = 1.19.^*$$

Prob. 15.—Calculate the integral in the preceding problem, using Simpson's rule, to see how closely the result checks with that obtained by the use of the trapezoid formula. Evaluate the same integral by the two methods, taking larger intervals—for instance, 10, and 15 degrees. This will show the advantage of Simpson's

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* The solution is also obtained directly through the Gamma function. According to formula 483 on page 62 of Peirce's table of integrals, we have

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta = \frac{1}{2}\sqrt{\pi}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)};$$

or, since

$$\Gamma(n+1) = n\Gamma(n),$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta = \frac{2}{3}\sqrt{\pi}\frac{\Gamma\left(\frac{7}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}.$$

Values of the common logarithms of $\Gamma(n)$ for values of n between 1 and 2 are tabulated on page 124 of Peirce's book.

rule over the straight-line formula, in that more accurate results are obtained with a smaller number of points.

Prob. 16. — In the solution of Prob. 14 and 15 it is assumed that the relation (8) holds for perfectly worn bearings, when the coefficient of friction is variable. Strictly speaking, this is not true, because Eq. (8) is deduced under the assumption of a constant coefficient of friction. Show that, when the coefficient of friction varies as in Prob. 14, the distribution of pressures in a perfectly worn bearing is that assumed in Prob. 6.

Solution. — The normal wear at a point is proportional to the work of friction per unit area. Since the velocity is the same at all points of the journal, the wear is proportional to the friction force per unit area. This force of friction is in our case $\mu p = \mu_1 \sqrt{vp}$, so that analogously with Prob. 3 we have

$$n = w \cos \theta = C\mu_1 \sqrt{vp},$$

where C is a constant. When $\theta = 0$ the foregoing equation gives

$$w = C\mu_1 \sqrt{vp_0}.$$

Dividing one by the other to eliminate C and w, gives $\cos \theta = \sqrt{\frac{p}{p_0}}$,

or $$p = p_0 \cos^2 \theta.$$

CHAPTER III.

FRICTION IN STEP BEARINGS.

A VERTICAL shaft *M* revolves on a step bearing *N* (Fig. 7); it is required to find the work lost in friction between the shaft and the bearing per one revolution of the shaft. This problem is similar to that of friction in horizontal journals, considered in Chapter II; only here it is necessary to determine the *work* of friction, while with horizontal journals it is sufficient to evaluate the *force* of friction. This difference is due to the fact that in a bearing supporting a horizontal shaft (Fig. 5) all the elementary forces of friction are at the same distance from the center of the shaft, and hence their points of application move with the same velocity; while in a step bearing the points of application of elementary forces of friction are moving at different velocities, from zero on the center line of the vertical shaft, to a maximum on its periphery. Therefore the forces cannot in this case be simply added together, but the share of each elementary force of friction must be determined in the

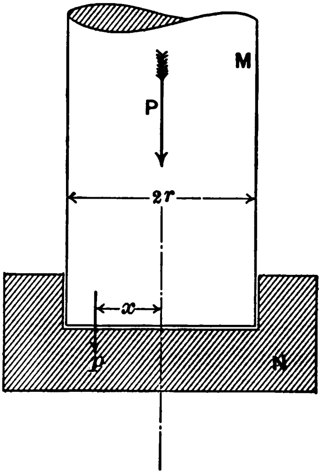

Fig. 7.—A flat step bearing supporting a vertical shaft.

proportion in which it contributes to the total resisting moment, or loss of energy.

The student is supposed to be familiar with the principal problems solved in Chapter II; therefore, some explanations are omitted to avoid repetition.

Prob. 1. — Deduce for a flat step bearing (Fig. 7) expressions for the work of friction, and pressure, analogous to Eq. (1), (2), and (3) of Chapter II.

Solution. — Let the pressure per square centimeter at a distance x from the center of the shaft be p; the total force of friction upon the infinitesimal zone $2\pi x \cdot dx$ is then $\mu p \cdot 2\pi x \cdot dx$, where μ is the coefficient of friction. The path traversed by this force during one revolution of the shaft is $2\pi x$, so that the work of friction per revolution, upon the infinitesimal zone, is

$$\mu p \cdot 2\pi x \cdot dx \cdot 2\pi x = 4\pi^2 \mu p x^2\, dx.$$

Total work of friction per revolution

$$W = 4\pi^2 \int_0^r \mu p x^2\, dx. \qquad (1)$$

If the coefficient of friction can be assumed to be independent of the velocity or pressure, the foregoing expression is simplified to

$$W = 4\pi^2 \mu \int_0^r p x^2\, dx. \qquad (2)$$

The unit pressures p must satisfy the condition that their sum over the entire bearing surface be equal and opposite to the total weight P resting upon the step bearing; or

$$P = \int_0^r p \cdot 2\pi x \cdot dx. \qquad (3)$$

As in the preceding chapter, these equations can be integrated only when the law of distribution of the pressures p is known or assumed. Some specific cases are considered in the problems that follow.

Prob. 2. — Find an expression for the work of friction in a flat step bearing, assuming the bearing to be new, that is to say, the pressures are distributed uniformly over the whole supporting area.

Solution. — The given condition is expressed by the equation

$$p = p_{av} = \text{Constant} \qquad (4)$$

Therefore Eq. (2) becomes

$$W = \mu p_{av} \cdot \frac{4\pi^2 r^3}{3},$$

and from Eq. (3) we find

$$P = p_{av} \cdot \pi r^2.$$

Eliminating p_{av} between these two expressions gives

$$W = \mu P \cdot \frac{4\pi r}{3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is the required formula for the work of friction per one revolution of the shaft. This formula can also be written in the form

$$W = \mu P \cdot 2\pi \left(\frac{2r}{3}\right)$$

which can be interpreted by saying that in a new bearing total friction may be considered as if concentrated at two-thirds of the radius of the shaft.

Prob. 3. — Find the law of distribution of pressure in a perfectly-worn flat step bearing; make use of the principle explained in Prob. 2, Chapter II.

Solution. — The required distribution of pressures is such as to cause the same vertical wear at all points of the bearing. But the wear is proportional to the work of friction per unit area (experimental fact); hence

$$\frac{\text{Work of friction}}{\text{Area}} = \frac{4\pi^2 \mu p x^2\, dx}{2\pi x\, dx} = \text{Constant},$$

or, after reduction, omitting the constant factors:

$$\mu p x = \mu_r p_r r = \text{Constant}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where μ_r and p_r are the values of the coefficient of friction and of the unit pressure at the periphery of the shaft. If the coefficient of friction is constant, the foregoing expression becomes

$$p x = p_r r = \text{Constant}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In words: the pressure in a worn step bearing varies inversely as the distance from the center of the shaft, being a minimum at the periphery. Eq. (7) probably does not hold good for points near the center of the shaft, because, when x approaches to zero, p approaches to infinity. There is no doubt, however, that in a worn step bearing the pressure near the center of the shaft is very great. In practice, the shaft always has a center hole for the purpose of centering the revolving part when turning it on a lathe, so that there is no pressure at the center, and therefore no objection to using formula (7).

Prob. 4. — Find an expression for the work of friction in a perfectly-worn flat step bearing.

Solution. — Integrating Eq. (2) and (3) with the use of Eq. (7), we find

$$W = \mu p_r \cdot 2\,\pi^2 r^3$$

and

$$P = p_r \cdot 2\,\pi r^2. \quad \text{. . . . .} \quad (8)$$

Eliminating p_r between these two equations gives

$$W = \mu P \cdot \pi r. \quad \text{. . . . .} \quad (9)$$

Eq. (8) and (9) can also be written in the form

$$p_r = \frac{1}{2}\left(\frac{P}{\pi r^2}\right) = \frac{1}{2}\,p_{av}, \text{ and } W = \mu P \cdot 2\pi\left(\frac{r}{2}\right).$$

The following conclusions can be drawn from these results, as compared with the results of Prob. 2: (*a*) The total work of friction is less in a worn bearing than in a new one, in the ratio of 3 to 4; (*b*) the resultant force of friction may be considered as if concentrated at one-half of the radius of the shaft; (*c*) the unit pressure p_r at the periphery of the shaft is equal to one-half of the pressure p_{av} obtaining in a new bearing; this means that the parts of the worn bearing near the center are subjected to considerably higher pressures than when the bearing is new.

Prob. 5. — The loss of power caused by friction in a step bearing can be expressed by the formula:

$$\text{Loss, in metric horse power, } = \frac{P \times D \times N}{\text{Constant}},$$

where P is the load upon the bearing in metric tons, D is the diameter of the shaft in meters, and N the speed in rev. per min. The constant is placed in the denominator, in order to make the formula more convenient for slide-rule computations. Determine the value of the constant in the foregoing formula for a worn flat step bearing, when the coefficient of friction $\mu = 0.03$.

Ans. $\text{Constant} = \dfrac{(2 \times 60 \times 75)}{(0.03 \times \pi \times 1000)} = 95.5$ (one metric horse power = 75 kg-m. per sec.).

Prob. 6. — Find expressions corresponding to (5) and (9) for the work of friction per revolution for a new flat step bearing, and also for a worn step bearing, when the shaft is hollow, the inside and the outside radii being r_1 and r_2.

$$\textit{Ans.}\quad W = \mu P \cdot \frac{4}{3}\pi\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$W = \mu P \cdot \pi\,(r_2 + r_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9')$$

When $r_1 = 0$ (solid shaft) these expressions become identical with (5) and (9).

Prob. 7. — Prove (*a*) by reasoning, (*b*) mathematically, that the work of friction is greater with a hollow shaft than with a solid shaft, provided that the total weight, and the average pressure per unit area, are the same in both cases.

Solution. — (*a*) The supporting area of the step bearing being the same in both cases, the outside radius of the hollow shaft must be larger than that of the solid shaft. Therefore, the average path described by elementary forces of friction during one revolution is greater in the hollow shaft; consequently, with a similar distribution of pressures, the work of friction is greater. (*b*) To prove that the value of W from expression (5′) is greater than that from Eq. (5), when $\pi r^2 = \pi\,(r_2^2 - r_1^2)$: We have

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = r_2 + \frac{r_1^2}{r_2 + r_1}.$$

But $r_2 > r$; hence

$$r_2 + \frac{r_1^2}{r_2 + r_1} > r.$$

A similar proof applies to Eq. (9) and (9′).

Prob. 8. — Prove that the work of friction in a perfectly-worn step bearing is expressed by $W = \mu_r p_r \cdot 2\,\pi^2 r^3$. *Hint.* — Substitute Eq. (6) into (1).

Prob. 9. — The expression for the work of friction, deduced in the preceding problem, contains the unknown pressure, p_r, at the periphery of the shaft. Show how to determine this pressure when the variable coefficient of friction is expressed by $\mu = \mu_1 \cdot \frac{v^n}{p^m}$, where μ, m, and n are given constants; v is the linear velocity at the point to which μ refers.

Solution. — Velocity v is proportional to the distance of the point from the center, so that Eq. (6) gives: $px \cdot \frac{x^n}{p^m} = \text{Constant}$, or

$x^{n+1} \cdot p^{1-m} = r^{n+1} \cdot p_r^{1-m}$. Physical conditions are such that p at the center is always larger than the average pressure, and theoretically approaches to infinity. Therefore, the foregoing equation has a practical meaning only when $m < 1$. Substituting the value of p into Eq. (3) and integrating, results in $P = p_r \cdot 2\,\pi r^2 \cdot \frac{(1-m)}{(1-2m-n)}$. From this equation the unknown pressure p_r can be determined. In order that the right-hand side of this equation be positive the condition must be fulfilled: $2m + n < 1$. When $m = n = 0$ the expression for P becomes identical with Eq. (8), because in this case $\mu = \mu_1 =$ Constant.

Note. — It may be objected that the expression for μ used in this problem becomes zero at the center of the shaft ($v = 0$); also that it approaches to zero with increasing pressure, and approaches infinity with decreasing pressure. A better formula for μ would be a modification of the form: $\mu = \mu_1 [v^n + a] \div \left[1 + b\frac{p^m}{(c+p)^m}\right]$. It gives finite values for μ within the range of pressures from zero to infinity; it gradually decreases to a finite value, different from zero, when the velocity v approaches to zero. However, the integration with this form of expression for μ would be much more involved; moreover, it would be hardly warranted in view of the meager information available with respect to the actual variations of the coefficient of friction in step bearings.

Prob. 10. — Assuming the unit pressure to vary inversely as the n-th power of the distance from the center of the shaft, prove that $p_r = p_{av} \cdot \frac{(2-n)}{2}$. *Hint.* — Substitute the value of p from the expression $px^n = p_r r^n$ into Eq. (3). The pressure $p_{av} = \frac{P}{\pi r^2}$. When $n = 1$, $p_r = \frac{1}{2}\, p_{av}$, a result already derived in Prob. 4.

Prob. 11. — Deduce expressions equivalent to Eq. (2) and (3) when the pivot, instead of being flat, is shaped according to a surface of revolution (Fig. 8).

Solution. — Let p be the normal unit pressure at a point distant x from the axis; consider the work of friction upon a zone of the surface having an infinitesimal width ds. Repeating the reasoning given in Prob. 1, we find

$$W = 4\,\pi^2 \mu \int_{r_0}^{r} px^2\,ds, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

the only difference, as compared with Eq. (2), being that the width ds is not the same as the increment dx of the radius. The condition of equilibrium of the shaft requires that the sum of the vertical projections of pressures p be equal to the weight P of the revolving part, or

$$P = \int_{r_0}^{r} p \cos \phi \cdot 2\,\pi x \, ds. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In order to be able to integrate these expressions, the law of distribution of pressures must be known and the shape of the surface of revolution given.

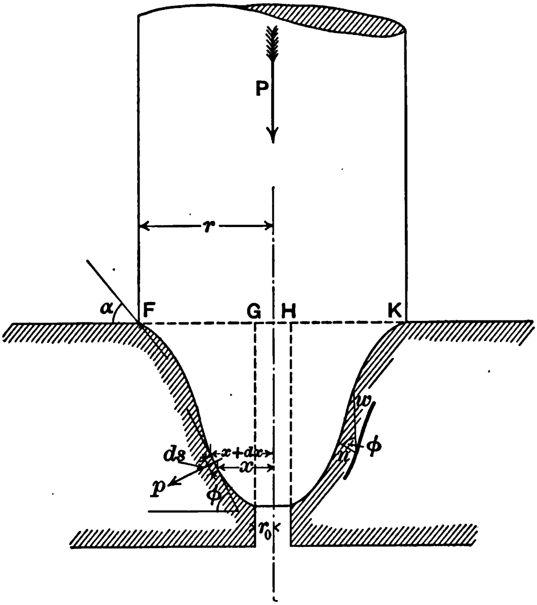

Fig. 8. — A pivot, shaped according to a surface of revolution.

Prob. 12. — Simplify Eq. (10) and (11) for the case of a new bearing, and show that the constant normal pressure upon the surface of revolution is equal to the average vertical pressure upon the projected area of the pivot.

Solution. —

$$W = 4\,\pi^2 \mu p_0 \int_{r_0}^{r} x^2 \, ds, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where the constant normal pressure p_0 is determined from the relation

$$P = 2\pi p_0 \int_{r_0}^{r} x \cos\phi \, ds. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But $ds \cos\phi = dx$, so that Eq. (13) integrated gives

$$P = p_0 \cdot \pi \, (r^2 - r_0^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The expression $\pi \, (r^2 - r_0^2)$ represents the projected area *FGHK* of the pivot (Fig. 8). Let the average vertical pressure on this area be p_{ver}. We have then $P = p_{ver} \cdot \pi \, (r^2 - r_0^2)$, and, comparing with Eq. (14), see that $p_0 = p_{ver}$.

Prob. 13. — Apply Eq. (10) and (11) to the case of a perfectly worn bearing, with a constant coefficient of friction.

Solution. — Vertical wear w (Fig. 8) must be constant at all points; but $w = \dfrac{n}{\cos\phi}$, where n is the wear normal to the surface. This latter wear is proportional to the work of friction per unit area, so that we find, as in Prob. 3, that n is proportional to the product px. Hence, the expression $\dfrac{px}{\cos\phi}$ must be constant,

or

$$\frac{px}{\cos\phi} = \frac{p_r r}{\cos\alpha} = B = \text{Constant}, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where B is the numerical value of the constant, and α is the angle which the tangent to the profile at the periphery of the shaft makes with the horizontal. Substituting the value of px from Eq. (15) into Eq. (10) and (11) gives

$$W = 2\pi\mu B \cdot \pi \, (r^2 - r_0^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$P = 2\pi B \int_{r_0}^{r} \cos^2\phi \, ds, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

or

$$P = 2\pi B \int_{r_0}^{r} \cos\phi \, dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17b)$$

In applications, either Eq. (17*a*) or (17*b*) is more convenient, according to the shape of the profile of the pivot.

Prob. 14. — Show that, when the surface of revolution is an ordinary truncated cone ($\phi = \alpha$ = Constant), the work of friction is expressed by

$$W = \left(\frac{\mu P}{\cos\alpha}\right) \cdot \frac{4}{3} \pi \frac{(r^3 - r_0^3)}{(r^2 - r_0^2)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

when the bearing is new, and by

$$W = \left(\frac{\mu P}{\cos \alpha}\right) \cdot \pi (r + r_0), \quad . \; . \; . \; . \; . \quad (19)$$

when the bearing is worn.

Prob. 15. — Show that Eq. (5) and (9) can be deduced as special cases of Eq. (18) and (19); also that friction loss is greater in a conical step bearing than in a flat one, with the same diameter of shaft.

Prob. 16. — Show that the work of friction in a new conical step bearing is greater than in the same bearing after it has been perfectly worn. *Hint:* $4(r^3 - r_0^3) - 3(r^2 - r_0^2)(r + r_0) = (r - r_0)^3 > 0$.

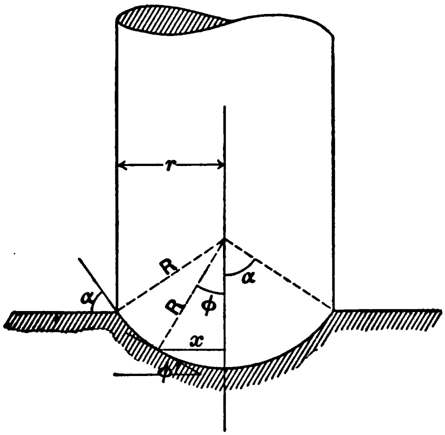

Fig. 9. — A spherical step bearing.

Prob. 17. — Prove that in a new spherical step bearing (Fig. 9) the work of friction is expressed by

$$W = 2\pi r \cdot \mu P \cdot \frac{\alpha - \sin \alpha \cos \alpha}{\sin^2 \alpha}. \quad . \; . \; . \; . \; . \quad (20)$$

Hint. — Substitute $x = R \sin \phi$, and $ds = R\, d\phi$ into Eq. (12) and (13).

Prob. 18. — Show that in a worn spherical step bearing the work of friction is

$$W = \mu P \cdot \frac{\sin \alpha}{\alpha + \sin \alpha \cos \alpha} \cdot 2\pi r, \quad . \; . \; . \; . \quad (21)$$

and that the unit pressure at a point defined by the central angle ϕ is expressed by

$$p = \frac{P}{\pi r^2} \cdot \frac{\cot \phi \sin^2 \alpha}{\alpha + \sin \alpha \cos \alpha} \cdot \quad . \ . \ . \ . \ . \ . \quad (22)$$

Hint. — Use Eq. (15) and the same substitution as in the preceding problem, into Eq. (10) and (11).

Prob. 19. — Eq. (20) and (21) must become identical with Eq. (5) and (9) when the radius R of the sphere becomes infinite, in other words, when $\alpha = 0$. Show that this is so by evaluating the indeterminate form $\frac{0}{0}$, which the equations assume when $\alpha = 0$.

Hint. — According to a well-known rule of the calculus, take the ratio of derivatives so that

$$\left[\frac{\alpha - \sin \alpha \cos \alpha}{\sin^3 \alpha}\right]_{\alpha=0} = \frac{0}{0} = \left[\frac{1 - \cos^2 \alpha + \sin^2 \alpha}{3 \sin^2 \alpha \cos \alpha}\right]_{\alpha=0}$$

$$= \left[\frac{2 \sin^2 \alpha}{3 \sin^2 \alpha \cos \alpha}\right]_{\alpha=0} = \left[\frac{2}{3 \cos \alpha}\right]_{\alpha=0} = \frac{2}{3}.$$

A similar procedure is applied to Eq. (21).

Prob. 20. — Assuming the work of friction in a worn flat bearing to be 100 per cent, plot a curve of values of the work of friction for worn spherical bearings for the same shaft, the radii R of the pivots varying from infinity (flat bearing) down to the radius r of the shaft (semi-spherical pivot).

Ans.			
	$\alpha = 0$	$\alpha = 45°$	$\alpha = 90°$
	$R = \infty$	$R = r\sqrt{2}$	$R = r$
	W = 100 per cent	W = 110 per cent	W = 127 per cent

Prob. 21. — In the step bearings considered above, the pressures, which are evenly distributed when a bearing is new, gradually become larger in the parts nearest to the center of the shaft. As a result, the bearing changes its shape, and wears out more quickly than it would, if the pressures could remain uniformly distributed, in spite of the wear. The problem is to find such a shape of the profile of the step bearing that the normal pressure is constant in the new, as well as in the worn bearing.

Solution. — Let p in Eq. (15) be constant; then the equation becomes

$$\frac{x}{\cos \phi} = \frac{r}{\cos \alpha} = \text{Constant} = T. \quad . \ . \ . \ . \ . \quad (23)$$

This is the equation of the profile of the bearing. The constant T represents the length of the tangent (Fig. 10), because $\frac{x}{\cos\phi}$ is the length of that part of the tangent to the curve, included between the point of contact and the axis of ordinates. Consequently, the required curve is defined by the condition that the length T of its tangent is constant. This property of the curve gives a simple method of construction. Let a_1 be a small weight with a pencil point fastened to it. Let OX and OY be axes of coördinates drawn on a horizontal board, and let the pencil point lie originally on the axis of abscissæ. Attach to the weight a piece of non-elastic string $a_1b_1 = T$ such that the end b_1 just reaches to the origin.

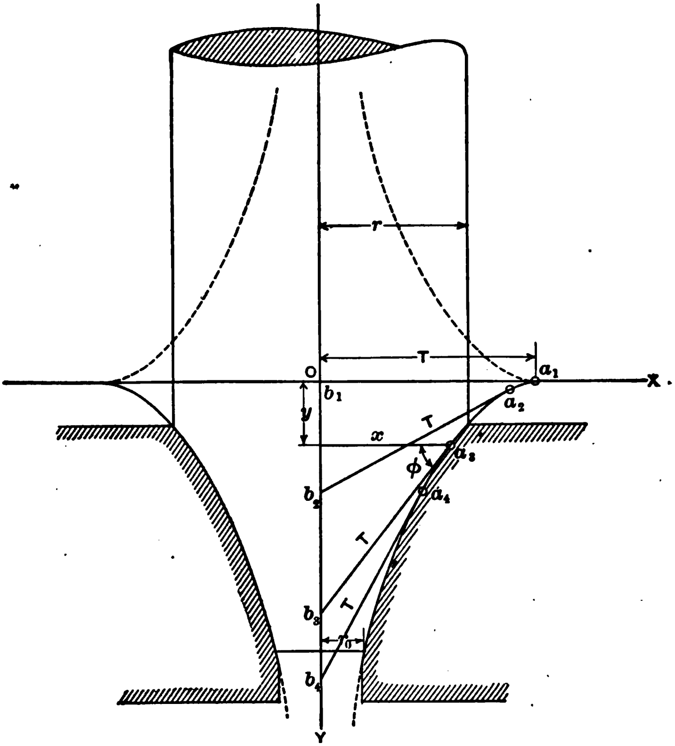

Fig. 10. — Schiele's pivot, the profile being a tractrix.

Draw the end of the string along the axis OY; the weight will follow, and the pencil will trace the desired curve, the tangent being at all points equal to the length T of the string. For the reason of this construction the curve is known as the *tractrix*.

The complete curve consists of four identical branches, which begin on the axis of abscissæ at the distances $\pm\ T$ from the origin and extend up and down approaching the axis of ordinates asymptotically. Any part of such a curve can be used as a profile for a step bearing, because all parts satisfy the condition (23) of a uniform distribution of pressure in spite of the wear.

Prob. 22. — A pivot, shaped according to the solution of the preceding problem, is known in engineering practice as *Schiele's anti-friction pivot.* Show that the work of friction in such a pivot, new or worn, is equal to

$$W = \mu P \cdot 2\pi T, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and determine which part of the tractrix should be selected for the profile in order to reduce the friction to a minimum.

Solution. — Eq. (24) is derived by using Eq. (23) and the condition $p =$ Constant in the general Eq. (10) and (11); these latter apply both to new and worn bearings. As shown by Eq. (24), the work of friction is a minimum when T is selected as small as possible. Eq. (23) and Fig. 10 show that it is best to select the part of the tractrix beginning with point a_1, at which the curve is tangent to the axis of abscissæ. T is then equal to the radius r of the shaft, and this is evidently the smallest value which can be assigned to T. The inner radius r_0 of the pivot is determined by the condition that the pressure p must not exceed a certain safe limit indicated by experience, for a particular service for which the shaft is designed.

Note. — A comparison of Eq. (24) with Eq. (9) shows that friction in Schiele's pivot is considerably greater than in an ordinary flat pivot; therefore the name "anti-friction" is somewhat a misnomer. It would be more appropriate to call Schiele's pivot "constant-pressure" or "minimum-wear" pivot.

Prob. 23. — Transform the Eq. (23) of the tractrix into one in rectangular coördinates (Fig. 10).

Solution. — $$x = T\cos\phi = \frac{T}{\sqrt{1+\tan^2\phi}} = \frac{T}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}},$$

from which $\frac{dy}{dx} = -\frac{\sqrt{T^2-x^2}}{x}$. The minus sign is selected, because the derivative is negative for the lower right-hand branch of the curve: y decreases when x increases. The integral of this equation is

$$y = T \operatorname{Ln}\left(\frac{T + \sqrt{T^2 - x^2}}{x}\right) - \sqrt{T^2 - x^2}. \quad . \quad . \quad (25)$$

The constant of integration = 0, because $y = 0$ when $x = T$. For purposes of calculation this equation can be expressed in a simpler form through hyperbolic functions, since tables of such functions are readily accessible.* Namely,

$$y = T\left\{\operatorname{Cosh}^{-1}\left(\frac{T}{x}\right) - \sqrt{1 - \left(\frac{x}{T}\right)^2}\right\}. \quad . \quad . \quad . \quad (26)$$

The coördinates of the tractrix can also be expressed conveniently through the angle ϕ. Substituting the value of $\frac{x}{T} = \cos\phi$ from Eq. (23) into Eq. (25), we get

$$\left.\begin{aligned} y &= T\left[\operatorname{Ln}\ \tan\left(\frac{\phi}{2} + \frac{\pi}{4}\right) - \sin\phi\right] \\ x &= T\cos\phi. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (27)$$

The curve can be constructed from these equations by assuming different values of ϕ.

Prob. 24. — Show that Eq. (25) and (26) can be represented in the form $\frac{y}{T} = f\left(\frac{x}{T}\right)$; this curve may be called the "generalized tractrix," good for any value of T. Plot this curve for values of $\frac{x}{T}$ from 1.00 to 0.05;† explain how to modify its ordinates and abscissæ in order to construct the profile of a Schiele's pivot, for a given T, and to a certain required scale.

Ans.	$\frac{x}{T} = 1.00$	0.50	0.05
	$\frac{y}{T} = 0$	0.451	2.69

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* Hyperbolic functions are being used to a considerable extent in physics and lately in engineering. The fundamental properties of these functions will be found in Seaver's *Mathematical Handbook* (McGraw-Hill), and in J. McMahon's *Hyperbolic Functions* (John Wiley & Sons).

† It is convenient to select different scales for abscissæ and for ordinates. However, in this case the length of the tangent is no longer constant.

CHAPTER IV.

CARRYING CAPACITY OF BELTS.

A CERTAIN amount of mechanical power is transmitted to the pulley *A* (Fig. 11) by means of the belt *BC*; *B* is the driving side of the belt, and *C* is the slack side.

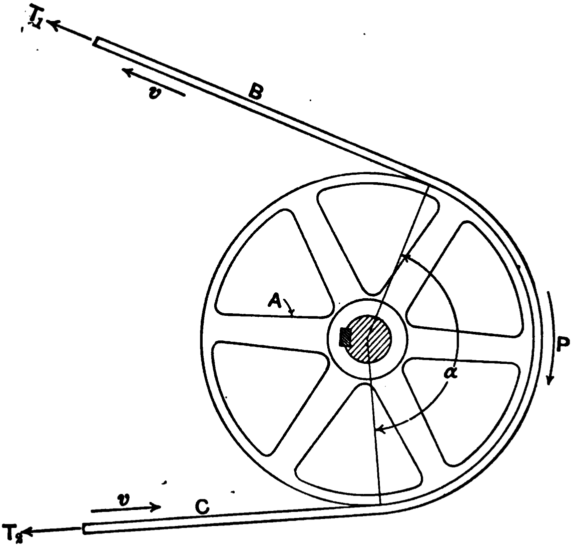

Fig. 11.—Belt and pulley.

The tension T_1 on the driving side is greater than the tension T_2 on the slack side, the difference between the two being the useful effort *P* transmitted to the pulley. This difference in tension is possible, because of friction between the belt and the pulley. The problem is to find the relation between T_1 and T_2, knowing the diameter of the pulley, the angle α of contact, and the coefficient of friction μ between the belt and the pulley.

Consider an infinitesimal element MN of the belt (Fig. 12) corresponding to the central angle $d\phi$. The forces acting upon this element are: tension T opposing the motion, a slightly larger tension $T + dT$ in the direction of the motion, and the tangential force of friction dF between the pulley and the belt. Since the element of the belt is in equilibrium, the force of friction balances the difference dT of the tensions T and $T + dT$. This fact expressed analytically gives the differential equation for the solution of the problem.

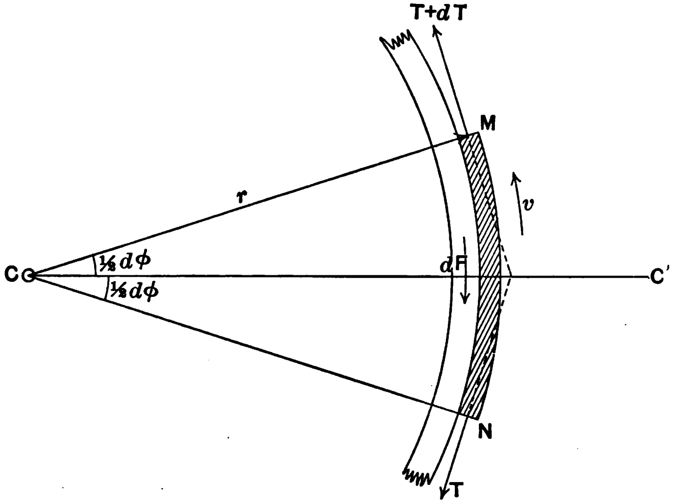

Fig. 12. — Forces acting upon an infinitesimal element of a belt

Friction between solid bodies is usually assumed to be proportional to the normal pressure. In our case, tension T gives in the direction $C'C$ a normal component $T \sin \frac{d\phi}{2}$ which presses the element of the belt to the pulley; tension $T + dT$ gives a component

$(T+dT)\sin\frac{d\phi}{2}$. Hence, the force of friction upon the infinitesimal element is

$$dF = dT = \mu\left[T\sin\frac{d\phi}{2} + (T+dT)\sin\frac{d\phi}{2}\right]. \quad . \quad (1)$$

Since we consider an infinitesimal element of the belt, the increment dT can be neglected in the last term, as compared to the tension T itself. Moreover, the sine of the angle $\frac{d\phi}{2}$ can be replaced by the angle itself (length of the arc of radius unity). Eq. (1) is then simplified to

$$dT = \mu T\,d\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or $\frac{dT}{T} = \mu\,d\phi$. Integrating this expression over the whole angle of contact, α, gives

$$\mathrm{Ln}\frac{T_1}{T_2} = \mu\alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which

$$\frac{T_1}{T_2} = e^{\mu\alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where e is the base of the natural logarithms. The useful effort exerted by the belt upon the pulley is

$$P = T_1 - T_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Eq. (4) and (5) are the necessary relations connecting the tensions T_1 and T_2 with the other quantities involved in belt drive.

Prob. 1. — In formula (3), angle α is in radians (why?); transform the formula into

$$\log\frac{T_1}{T_2} = \frac{\alpha}{\text{const.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where α is in degrees, and $\mu = 0.27$.

Ans. Const. $= \frac{2.3 \times 180}{0.27\,\pi} = 488.$

Prob. 2. — Express the tensions in the driving side and in the slack side of the belt, as functions of the useful effort P.

Ans. Eq. (4) and (5) give

$$T_1 = P \cdot \frac{e^{\mu\alpha}}{e^{\mu\alpha} - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$T_2 = P \cdot \frac{1}{e^{\mu\alpha} - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Prob. 3. — A belt has to transmit a useful effort of 95 kg.; the angle of contact is 160 degrees, the coefficient of friction is 0.30. What is the tension on the driving side, and how wide does the belt have to be, if the permissible safe tension is 7 kg. per cm. of its width? *Ans.* 167 kg.; 24 cm.

Hint. — Use formula (7).

Prob. 4. — Check the solution of the preceding problem by using directly Eq. (5) and (6). This method has some advantage because it is not necessary to evaluate the exponential expression $e^{\mu\alpha}$. Note that the constant in formula (6) must be modified to suit the given value of the coefficient of friction.

Prob. 5. — When one has to calculate regularly a large number of belts, using Eq. (7), it is convenient to prepare a table or plot a curve, giving the values of the exponential expression by which P is to be multiplied in order to obtain T_1. Plot such a curve for $\mu = 0.28$ and for angles of contact from 90 to 270 degrees.

Ans. $\alpha =$	90°	180°	270°
$\frac{T_1}{P} =$	2.81	1.71	1.36

Prob. 6. — Express Eq. (7) and (8) through hyperbolic functions.

Solution. —

$$\frac{e^{\mu\alpha}}{e^{\mu\alpha} - 1} = \frac{e^{\frac{1}{2}\mu\alpha}}{e^{\frac{1}{2}\mu\alpha} - e^{-\frac{1}{2}\mu\alpha}} = \frac{\text{Cosh}\,\frac{1}{2}\mu\alpha + \text{Sinh}\,\frac{1}{2}\mu\alpha}{2\,\text{Sinh}\,\frac{1}{2}\mu\alpha},$$

so that

$$T_1 = \frac{P}{2}(\text{Coth}\,\tfrac{1}{2}\mu\alpha + 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Analogously,

$$T_2 = \frac{P}{2}(\text{Coth}\,\tfrac{1}{2}\mu\alpha - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Prob. 7. — Check the solution of Prob. 3 by means of formula (9).

Prob. 8. — When the peripheral speed of the belt is considerable, centrifugal force acting upon it tends to separate it from the pulley.

Consequently, the normal pressure, and the friction between the belt and the pulley, are reduced, as well as the maximum effort which the belt can transmit without slipping. Deduce a relation between T_1 and T_2, analogous to Eq. (3), but in which centrifugal forces are taken into account.

Solution. — According to the general laws of circular motion, centrifugal force $= \frac{\text{mass of the body} \times (\text{velocity})^2}{\text{radius of rotation}}$. Let w be the weight of one meter of belt, in kilograms; then the centrifugal force acting upon the element of the belt, corresponding to angle $d\phi$ (Fig. 12), is $\left(\frac{w}{g}\cdot r\,d\phi\right)\cdot\frac{v^2}{r} = \frac{w}{g}\,v^2 d\phi$, whe e $g = 9.81$ is the acceleration due to gravity (in meters per sec.^{2}), r is the average radius of curvature of the belt on the pulley, in meters, and v the linear velocity of the belt, in meters per second. This centrifugal force reduces the normal pressure upon the pulley, so that Eq. (2) becomes

$$dT = \mu\left(T - \frac{wv^2}{g}\right)d\phi.$$

This equation, after being integrated similarly to Eq. (2), gives

$$\text{Ln}\left(\frac{T_1 - \frac{wv^2}{g}}{T_2 - \frac{wv^2}{g}}\right) = \mu\alpha. \quad . \; . \; . \; . \; . \; . \quad (11)$$

When the speed is low so that the correction term $\frac{wv^2}{g}$ can be neglected, Eq. (11) becomes identical with (3).

Prob. 9. — Deduce formulæ analogous to (7) and (8), taking the centrifugal force into consideration.

Ans. Writing Eq. (11) in the exponential form and solving with Eq. (5), we get

$$T_1 = P\cdot\frac{e^{\mu\alpha}}{e^{\mu\alpha}-1} + \frac{wv^2}{g}, \quad . \; . \; . \; . \; . \; . \quad (12)$$

and

$$T_2 = P\cdot\frac{1}{e^{\mu\alpha}-1} + \frac{wv^2}{g}. \quad . \; . \; . \; . \; . \; . \quad (13)$$

Prob. 10. — Solve Prob. 3, taking the centrifugal force into consideration. Assume $v = 16$ m. per sec., and the unit weight of the belt = 7.3 kg. per sq. m.

Solution. — Let the unknown width of the belt be x cm.; then the tension $T_1 = 7\,x$, and the weight per meter of length, $w = \frac{7.3\,x}{100}$. Substituting these values into Eq. (12) and using the answer of Prob. 3, we obtain: $7\,x = 167 + 1.9\,x$, whence $x = 32.7$ cm. Tension $T_1 = 7\,x = 229$ kg.

Prob. 11. — With what original tension must a belt be put on its pulleys in order to transmit a useful effort P? Disregard the action of the centrifugal force.

Solution. — The original tension T_0 on either side of the belt is an arithmetical mean of the working tensions T_1 and T_2, so that from Eq. (7) and (8)

$$T_0 = \frac{P}{2} \cdot \frac{(e^{\mu\alpha} + 1)}{(e^{\mu\alpha} - 1)}, \quad \ldots \ldots \ldots \quad (14)$$

or, in hyperbolic functions, according to Eq. (9) and (10),

$$T_0 = \frac{P}{2} \cdot \text{Coth}\, \tfrac{1}{2}\, \mu\alpha. \quad \ldots \ldots \ldots \quad (15)$$

Note. — The sum of the tensions T_1 and T_2 remains constant with variations in load, and is equal to the total original tension $2\,T_0$. This is because tensions are proportional to elongations, and the extra elongation on the driving part must be compensated for, by a reduced elongation on the slack side.

Prob. 12. — What original tension should be given to the belt considered in Prob. 3, in order that the belt could stand 25 per cent overload, without slipping on the pulley?

Ans. About 150 kg.

Prob. 13. — Correct formula (14) for the effect of centrifugal force.

Ans. $$T_0 = \frac{1}{2}(T_1 + T_2) = \frac{P}{2} \cdot \frac{(e^{\mu\alpha} + 1)}{(e^{\mu\alpha} - 1)} + \frac{wv^2}{g}. \quad \ldots \ldots \quad (16)$$

This result shows that, on account of centrifugal force, it is necessary to give the belt a larger original tension. When running at a speed v the part $\frac{wv^2}{g}$ of the tension is balanced by the centrifugal force, and only the rest is effective in producing friction upon the pulley, with a consequent turning effort.

Prob. 14. — What should be the original tension of the belt in Prob. 10, in order that it would not slip at 25 per cent overload?

Ans. $150 + \frac{7.3 \times 0.327 \times 16^2}{9.81} = 212$ kg.

Prob. 15. — The belt considered in Prob. 10 is put on its pulleys with a tension of 200 kg. per side, at rest. Plot a curve showing the maximum useful effort P which can be transmitted with this belt at different speeds. Determine the *critical speed* at which the centrifugal force balances the whole of the original tension, so that the belt does not press upon the pulley, and therefore exerts no useful tangential effort.

Ans. Eq. (16) becomes: $P = \dfrac{(200 - 0.2434\, v^2)}{1.263}$

The following points lie on this curve:

$v =$ 0	10	20	28.7
$P =$ 158	139	81.3	0

The last point corresponds to the critical speed.

Prob. 16. — Plot on the same curve sheet with that of the preceding problem, a curve of the useful power, in kilogram-meters per second, transmitted by the belt.

Ans. Power = Pv, so that when

$v = 0$	10	20	28.7
Power = 0	1390	1626	0

Prob. 17. — The result of the preceding problem shows that the power transmitted by the belt reaches its maximum at a certain speed, and decreases with a further increase in speed, becoming zero at the critical speed. Show that the speed corresponding to maximum power is equal to $\dfrac{1}{\sqrt{3}}$ of the critical speed, and that at this speed one-third of the original tension is taken up by the centrifugal force. *Hint.* $v\left(T_0 - \dfrac{wv^2}{g}\right)$ reaches its maximum when

$$v = \sqrt{\frac{T_0 g}{3\, w}}; \quad \text{Critical speed} = \sqrt{\frac{T_0 g}{w}}.$$

Prob. 18. — Check the point of maximum power, obtained in Prob. 16, with the formula deduced in Prob. 17.

Ans. 1747 kg. at 16.55 m. per sec.

Prob. 19. — Supplement the curves obtained in Prob. 15 and 16 with curves of tensions T_1 and T_2.

Ans.	$v =$ 0	10	20	28.7
	$T_1 =$ 279	269.5	240.6	200
	$T_2 =$ 121	130.5	159.4	200

Prob. 20. — The results of the preceding problem show that the tension T_1 decreases, while T_2 increases with speed, when the belt transmits its maximum tractive effort; at the critical speed $T_1 = T_2 = T_0$. Prove these results in the general form, and show that the curves of T_1 and T_2 are parabolæ, one turned downward, the other upward. *Hint.* — Eliminate P from Eq. (12) and (13) by means of formula (16), and use the critical speed from Prob. 17.

CHAPTER V.

TORSION OF SHAFTS.

Some mechanical power is transmitted from pulley *A* (Fig. 13) through the shaft *B* to pulley *C*. The pulley *C* resists the rotation, so that the shaft is subjected to two equal and opposite turning efforts, those of *A* and of *C*; hence the shaft is twisted, or, in engineering language, is said to be subjected to torsion. It is required to find a relation between the turning moment acting on the pulleys and the safe size of the shaft.

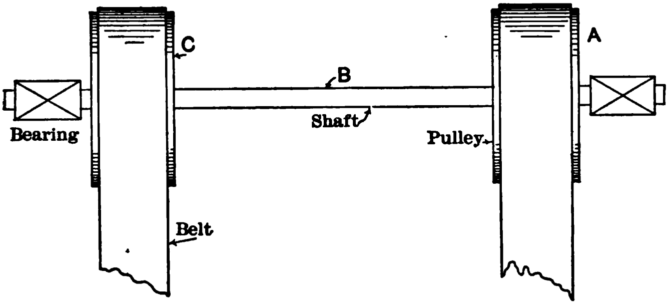

Fig. 13. — A shaft and two pulleys.

For the purposes of mathematical analysis, the twisting of the shaft is considered with reference to its consecutive cross sections. Each cross section is turned by an infinitesimal angle relatively to the next cross section, bringing into play the elastic reactions of the material. According to the fundamental law of equilibrium, the reaction is equal to the action, so that the resultant moment of these elastic forces in any cross section of the shaft is equal and opposite to the turning moment applied to the driving pulley. This law makes it possible to establish a relation between the applied moment and the elastic stresses in the material.

Let Fig. 14 represent a cross section of the shaft, and let q be the resisting force in the material, in kilograms per one square centimeter of cross section, at a distance of x centimeters from the center of the shaft. This unit force is called the shearing stress in the material, since it is caused by the tendency of one cross section to slide with respect to the next cross section. The shearing force upon an infinitesimal annulus of the width dx is $q \cdot 2\pi x \cdot dx$, and the moment of this force, with respect to the axis of the shaft, is $q \cdot 2\pi x \cdot dx \cdot x$. The above stated condition of equilibrium becomes, therefore,

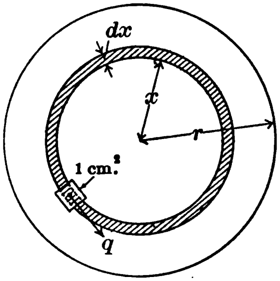

Fig. 14. — Cross-section of a shaft subjected to torsion.

$$M = 2\pi \int_0^r q\, x^2\, dx, \quad \ldots \ldots \quad (1)$$

where M is the applied turning moment. In this equation q is a variable quantity, so that, in order to perform the integration, q must be given as a function of the distance x from the center.

According to the fundamental assumption of the theory of elasticity, stresses are proportional to the corresponding strains or displacements (as long as a certain limit, called the elastic limit of the material, is not exceeded). In our case, assuming that each cross section turns as a whole with respect to the next cross section, displacements are the largest at the periphery, and decrease to zero at the center. The angle of turning being the same for all the points of a cross section, actual linear displacements are proportional to the distances x from the center. We have thus, remembering that stresses are proportional to displacements,

$$\frac{q}{q_r} = \frac{x}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where q_r is the stress at the periphery of the shaft. This stress is larger than the stress at any other point of the cross section, so that if q_r does not exceed the safe limit, determined by experience, the whole shaft is safe with respect to torsion. Substituting the value of q from (2) into (1), we obtain

$$M = \frac{2\pi q_r}{r} \cdot \int_0^r x^3 dx,$$

or

$$M = \frac{q_r \pi r^3}{2} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the required relation between the turning moment M and the maximum stress q_r in the shaft. If q_r is in kilograms per square centimeter, and r is in centimeters, the applied moment M must be expressed in kilogram-centimeters.

Prob. 1. — Draw a curve giving values of safe twisting moments for shafts up to 30 cm. in diameter; assume the safe permissible stress to be 600 kg. per sq. cm. What is the mathematical name of the curve?

Ans. $M = 31{,}800$ kg-m. for $D = 30$ cm.; cubic parabola.

Prob. 2. — The diameter of the shaft can be expressed from formula (3) as

$$D = k_1 \sqrt[3]{M}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where k_1 is a constant. Calculate k_1 for $q_r = 800$ kg. per sq. cm., so that D be expressed in centimeters when M is in kilogram-meters.

Ans. $k_1 = \sqrt[3]{\frac{16 \times 100}{800\pi}} = 0.86.$

Prob. 3. — In practice, the power P transmitted through the shaft, and the speed N of rotation are usually given instead of the turning moment M. Expression (4) becomes, then,

$$D = k_2 \sqrt[3]{\frac{P}{N}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

where k_2 is another constant. Express k_2 through k_1, when P is in kilowatts, and N in rev. per min. (1 kg-m. per sec. = 9.81 watts.)

$$\textit{Ans.} \quad P = \frac{9.81}{1000} \cdot M \cdot \frac{2\pi N}{60} = 1.02 \frac{MN}{1000};$$

hence

$$k_2 = \frac{10\,k}{\sqrt[3]{1.02}} = 10\ k_1 \text{ (nearly).}$$

Prob. 4. — How is formula (3) modified when the shaft is hollow, the inside radius being r_0?

Solution. —

$$M = \frac{2\pi q_r}{r} \int_{r_0}^{r} x^3\,dx = \frac{\pi q_r}{2r} \cdot (r^4 - r_0^4), \quad . \quad . \quad . \quad . \qquad (6)$$

or

$$M = q_r \cdot \frac{\pi r^3}{2} \cdot (1 - n^4), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

where $n = \frac{r_0}{r}$. For a solid shaft $n = 0$, and Eq. (7) becomes identical with (3).

Prob. 5. — A hollow shaft has the inside diameter equal to one-half of the outside diameter. What is the saving in material, as compared with a solid shaft having the same outside diameter, and what is the corresponding loss in strength?

Ans. Saving in material, $s = n^2 = 0.25 = 25$ per cent; loss in safe resistance to torsion, $l = n^4 = 0.0625 = 6.25$ per cent.

Hint. — The volume of a hollow shaft per centimeter length is expressed by

$$\pi\,(r^2 - r_0^2) = \pi r^2\,(1 - n^2) \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Prob. 6. — Extend the solution of the preceding problem to values of n from zero to unity, and plot the results in the form of curves, against values of n as abscissæ.

Ans.	n	s	l
	0	0	0
	0.5	25	6.25
	1.0	100	100

Prob. 7. — Referring to the preceding problem, at what value of n is the difference between the numerical values of s and l a maximum?

Ans. $n = \frac{\sqrt{2}}{2} = 0.707.$

Hint. — Equate to zero the first derivative of the expression $(n^2 - n^4)$.

Prob. 8. — Show that the solution of Prob. 7 corresponds to the value of n at which the tangents to the two curves (Prob. 6) are parallel to each other.

Prob. 9. — Two shafts are given, of equal length and weight, and are made of the same material. One shaft is solid, the other shaft is hollow and has the inside diameter equal to two-thirds of the outside diameter. How much stronger is the second shaft? Draw the cross sections of the two shafts to the same scale, for a comparison.

Ans. 1.94 times stronger.

The ratio of the outside diameters is $\frac{3}{\sqrt{5}} = 1.342.$

Prob. 10. — Find the value of n at which a hollow shaft has the greatest resistance to twist, per unit weight.

Solution. — From Eq. (7) and (8) we see that the ratio

$$\frac{r^3(1 - n^4)}{r^2(1 - n^2)} = r(1 + n^2) \quad . \; . \; . \; . \; . \; . \; . \quad (9)$$

must be a maximum. This expression increases indefinitely as n and r increase; the maximum value of n is unity, and the maximum value of r is infinity, so that theoretically the shaft of maximum resistance per unit weight is a tube of an indefinitely large radius, with infinitesimally thin walls. This result could have been foreseen from the theory of torsion, given above; only the material at the periphery of the shaft is stressed to its safe limit, so that, in order to get the full advantage of the material, all of it must be concentrated at the periphery, or as near to it as possible. Moreover, the periphery must be as far as possible from the center, in order to increase the resisting moment of the stresses with respect to the axis of the shaft (see Fig. 14). Hence, in practice, hollow, thin-walled shafts, of as large a diameter as is feasible, approach the condition of maximum strength with a minimum weight.

Prob. 11. — Let the requirement of the preceding problem be limited by the condition that the thickness of the wall of the hollow shaft must be not less than a certain practical limit of t centimeters. Is the result modified thereby?

Solution. — The new condition is expressed by: $r(1-n) = t$. Substituting this value of r into Eq. (9), we find that the expression

$$y = \frac{(1+n^2)}{(1-n)} \qquad (10)$$

must be a maximum. This expression increases with n, and reaches its largest numerical value, equal to infinity, at $n = 1$. But $n = 1$, with a finite thickness of the wall, means that the radius of the shaft must be infinite. Consequently, a hollow shaft, with a given thickness of the wall, is the stronger per unit weight, the larger its radius. This result could have also been foreseen directly from the theory of torsion.

Prob. 12. — Suppose that the student did not see directly that y in Eq. (10) reaches its largest numerical value at $n = 1$. Proceeding, then, according to the rule of the calculus, that is to say, equating the first derivative of y to zero, he would find two solutions, *viz.*, $n = 1 \pm \sqrt{2}$. Plot curve (10) between the limits $n = -1$ and $n = +3$, and explain the meaning of the three solutions for n, determining, if necessary, the sign of the second derivative.

Prob. 13. — A hollow shaft has to withstand a given torsional moment and be as light as possible. Find its external and internal radii.

Solution. $r^2(1-n^2)$ must be a minimum, with the limiting condition that $r^3(1-n^4)$ is constant. This latter condition can be written in the form: $r^2 \cdot r \cdot (1-n^2)(1+n^2) = \text{Constant}$, so that

$$r^2(1-n^2) = \frac{\text{Constant}}{r(1+n^2)} = \text{minimum},$$

or $r(1+n^2) = \text{maximum}$. This is identical with the condition (9), so that the solution of Prob. 10 applies in this case.

Prob. 14. — The cost of making a shaft increases with its diameter, especially in large sizes; therefore the weight is not the only factor which determines the cost of the shaft. Only the weight has been taken into consideration in Prob. 10 to 13. In order to see how the cost of manufacture can be taken into account, assume that the price for rolling, per centimeter of length, is expressed by the formula $a + br$ cents, where a and b are empirical constants. The price of steel is p cents per cubic centimeter. Find under these conditions the dimensions of a hollow shaft which must stand a given torsional moment, and the cost of which, per unit length, is a minimum.

Solution. — Eq. (7) gives

$$r^3(1-n^4)=\frac{2M}{\pi q_r}=\text{Constant.} \quad . \quad . \quad . \quad . \quad (11)$$

The total cost of making the shaft, per centimeter of length, including the cost of the material, must be a minimum, or

$$p\cdot\pi r^2(1-n^2)+a+br=\text{minimum.} \quad . \quad . \quad (12)$$

The usual method of solving equations like (11) and (12) consists in eliminating one of the independent variables, n or r, and differentiating with respect to the other variable. It is simpler, however, in this particular case, to differentiate both equations first. Namely, the complete differential of expression (11) is equal to zero, because the expression itself is equal to a constant; the complete differential of expression (12) is equal to zero for the values of n and r which convert it into a minimum. Thus we have

$$3r^2dr(1-n^4)-r^3\cdot 4n^3dn=0;$$
$$2r\cdot dr(1-n^2)-r^2\cdot 2n\,dn+b'dr=0,$$

where a new constant b' is introduced for the sake of abbreviation:

$$b'=\frac{b}{\pi p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Eliminating dn and dr from the foregoing two expressions gives, after reduction,

$$\frac{2b'}{r}=\frac{3}{n^2}+n^2-4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The required dimensions of the shaft are found by solving together Eq. (11) and (14).

Prob. 15. — Apply the solution of the preceding problem to the following practical case: A propeller shaft for a large steamer must be designed for a maximum twisting moment of 255,000 kg-m., the limiting stress q_r not to exceed 250 kg. per sq. cm. The price of steel is 40 dollars per metric ton, and the rolling mills charge 25.30 dollars extra, per meter length of shaft, for each centimeter of increase in diameter, above a certain size.

Solution. — The constant in Eq. (11) has the value of

$$\frac{2\times 255{,}000\times 100}{250\,\pi}=64{,}900\text{ cu. cm.};$$

$$b = \frac{25.30 \times 100}{2 \times 100} = 12.65 \text{ cents per cm. radius, per cm. length;}$$

$$p = \frac{40 \times 100 \times 7.8}{1000 \times 1000} = 0.0312 \text{ cents per cu. cm.,}$$

where 7.8 is the specific gravity of steel. Therefore,

$$b' = \frac{12.65}{0.0312\pi} = 129 \text{ sq. cm.}$$

The values of n and r are found either by trials, or by plotting the curves represented by Eq. (11) and (14), and determining the point of their intersection.

Ans. $r = 41.5$ cm.; $n = 0.55$.

Prob. 16. — Check the solution of the preceding problem by plotting curve (12) of the cost of the shaft. Take values of n between zero and 0.90, and the corresponding values of r found from Eq. (11). For the same values of r and n, draw also a curve of the weight of the shaft per centimeter length. Plot both curves on the same sheet, against values of n as abscissæ.

Ans.

n, %	r, cm.	Weight, kg.	Cost, dollars
0	40.19	39.59	$6.68 + a$
55	41.50	29.43	$6.40 + a$ (minimum)
90	57.36	15.30	$7.87 + a$

Note. — The curves show that, in spite of wide differences in weight, the total cost remains practically constant within a wide range of suitable values for n and r. This is because the saving in material with larger radii is nearly compensated for by a higher cost of rolling. Numerous other conditions, which usually enter into any engineering problem, reduce this range of radii to much narrower limits.

CHAPTER VI.

MOMENT OF INERTIA OF FLYWHEELS.

In many machines, such as steam and gas engines, pumps, punches, etc., the effort on the shaft varies periodically within wide limits. At the same time, the speed of rotation must be kept constant within narrow limits. This is accomplished by mounting on the shaft a heavy wheel (Fig. 15) — so-called *flywheel* — which, on account of its large inertia, tends to smooth fluctuations of speed. At moments when the driving power exceeds the power actually spent in useful work and

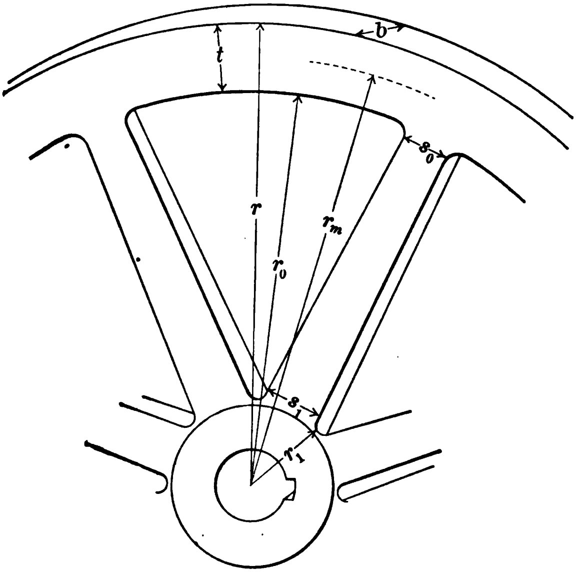

Fig. 15. — A portion of a flywheel.

friction, the surplus of energy accelerates the machine, including the flywheel. But it takes a considerable amount of energy to accelerate the heavy flywheel; therefore, the same surplus of energy changes the speed of the machine by a smaller amount than it would without the flywheel. When, at a next moment, the driving power falls below the required power, the machine and the flywheel slow down, giving up a part of their stored energy. Here, again, a comparatively small reduction in speed is sufficient to liberate a considerable momentum, stored in a heavy flywheel.

In brief, the flywheel acts as a storehouse of energy, periodically absorbing and releasing it through small variations in speed. It must be clearly understood, however, that a flywheel is not in itself a source of energy; therefore, the *average* driving power must be equal to the average power demanded of the machine, otherwise the machine will either gradually speed up or slow down in spite of the flywheel.

The dimensions of a flywheel are determined by the condition that, with a given range of fluctuation of power, the variations in speed should not exceed a given limit. It is therefore necessary to know how to determine the amount of energy stored in a flywheel of certain dimensions and weight, revolving at a given speed.

Theory and experiment show that the energy possessed by a material point of weight p, moving at a velocity s, is $\frac{ps^2}{2g}$, where g is the acceleration due to gravity. If p is in kilograms, and the speed s in meters per second, the energy is in kilogram-meters; the acceleration of gravity $g = 9.81$ meters per sec^2. The foregoing expression for energy is true for rectilinear motion and for a circular path. Hence, the energy possessed by a flywheel is

$$E = \frac{1}{2g} \cdot \int s^2 \, dp, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where dp is the weight of an infinitesimal element of the flywheel, moving at a velocity s. The integration is extended over the whole volume of the wheel.

Expression (1) can be transformed as follows: Let the flywheel revolve at a speed of N revolutions per minute; then the velocity of a point, distant x meters from the center, is, in meters per second,

$$s = 2\pi x \cdot \frac{N}{60}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Further, if p_1 is the weight of one cubic meter of the material of the flywheel, we have also

$$dp = p_1 \, dv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where dv is the corresponding element of the volume.

Substituting (2) and (3) into (1) gives

$$E = \frac{1}{2g} \cdot \left(\frac{2\pi N}{60}\right)^2 \cdot p_1 \int x^2 \, dv. \quad . \quad . \quad . \quad (4)$$

Apart from the constant multiplier $\frac{1}{2g}$, this expression consists of two factors, one depending only upon the speed of the flywheel, the other only upon its shape and material. This latter factor, which properly characterizes the flywheel itself, with respect to its capacity for storing energy, is called its *moment of inertia.* Thus,

$$\text{Moment of inertia} = p_1 \int_0^v x^2 \, dv. \quad . \quad . \quad . \quad (5)$$

Since p_1 is only a constant by which the results of integration are multiplied, we shall confine ourselves chiefly to the *geometric* moment of inertia, denoted here by I. That is

$$I = \int_0^v x^2\,dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

I is expressed in meters to the fifth power (m.^{5}), when the volume is in cubic meters, and x is in linear meters. The object of this chapter is to show how to calculate I in some important practical cases.

Prob. 1. — A hollow brass cylinder, with walls 5 mm. thick, and of an average radius 0.5 m., revolves around its geometric axis, at a speed of 100 rev. per min. How much energy is stored in the cylinder, per centimeter of its length? Specific gravity of brass is 8.55. Neglect the small amount of energy stored in the light spokes which fasten the cylinder to the shaft.

Solution. — The thickness of the cylinder is so small, as compared with its radius, that the whole volume can be assumed, without an appreciable error, to be concentrated at the average radius of the cylinder. In other words, x in Eq. (6) can be taken as constant and equal to 0.5 m. The volume of the cylinder per centimeter length is very nearly equal to $2\pi \times 0.5 \times 5 \times 10^{-3} \times 10^{-2} = 15.708 \times 10^{-5}$ cu. m. Consequently, the geometric moment of inertia $I = \overline{0.5}^2 \times 15.708 \times 10^{-5} = 3.927 \times 10^{-5}$ m^5. Then, according to Eq. (4),

$$E = \left(\frac{1}{19.62}\right) \times \left(\frac{2\pi \cdot 100}{60}\right)^2 \times (1000 \times 8.55) \times 3.927 \times 10^{-5}$$
$$= 18.77 \text{ kg-m.}$$

Prob. 2. — Deduce from the solution of the preceding problem, a general expression for the moment of inertia of a thin cylinder.

$$\textit{Ans.} \quad I = 2\pi bt \cdot r_m^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where t is the thickness of the rim, b its width, and r_m the average radius (Fig. 15).

Prob. 3. — Find a more general expression than the one given above, for the moment of inertia of a cylinder, when its thickness t is not small as compared with the average radius.

Solution. — The cylinder can be considered as consisting of infinitely thin concentric shells; the volume of an infinitesimal shell is $dv = 2\pi x \cdot dx \cdot b$, where b is the width of the cylinder, parallel to the axis. Substituting this expression for dv into Eq. (6) and integrating, gives

$$I = 2\pi b\int_{r_0}^{r} x^3\,dx = \frac{\pi b}{2}\cdot(r^4 - r_0^4), \quad . \; . \; . \; . \quad (8)$$

where r is the outside radius, and r_0 the inside radius of the cylinder.

Prob. 4. — Show that with the data given in Prob. 1, the moment of inertia calculated from formula (7) is practically the same as calculated by means of the more accurate expression (8).

Note. — In applying formula (8), use tables of logarithms (not a slide rule) and be very careful about the last decimals.

Ans. $I = \frac{\pi}{200}\cdot(\overline{0.5025}^4 - \overline{0.4975}^4)$.

Prob. 5. — The answer to the preceding problem shows that formula (8) is inconvenient for numerical computations when the thickness of the cylinder is small compared with its diameter. Express (8) through the outside radius r and the thickness t of the rim, so as to avoid computing a difference of two quantities r^4 and r_0^4 nearly equal to each other.

Solution. —

$$r_0 = r - t; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

hence $r^4 - r_0^4 = r^4 - r^4 + 4r^3t - 6r^2t^2 + 4rt^3 - t^4$;

or, after reduction,

$$I = 2\pi bt\cdot r^3\left[1 - 1.5\left(\frac{t}{r}\right) + \left(\frac{t}{r}\right)^2 - 0.25\left(\frac{t}{r}\right)^3\right] . \; . \; . \quad (10)$$

The terms within the brackets converge rapidly with small ratios of $\frac{t}{r}$; in most practical cases the first two terms are the only ones to be considered.

Prob. 6. — Check the solution of Prob. 4 by means of formula (10), and show the advantage of the latter in numerical computations. How many terms in formula (10) are needed in this particular case?

Prob. 7. — What is the largest value of the ratio $\frac{t}{r}$ with which the last two terms in formula (10) can be omitted, provided the error must be not over one per cent?

Solution. — From the condition

$$\left(\frac{t}{r}\right)^2_{\max} = 0.01\left[1 - 1.5\left(\frac{t}{r}\right)_{\max} + \left(\frac{t}{r}\right)^2_{\max}\right],$$

we find $\left(\frac{t}{r}\right)_{\max} = 0.0932$.

Prob. 8. — Extend the solution of the preceding problem to values of error from zero to 10 per cent, and plot the results in the form of a curve.

$$\textit{Ans.}\quad \left(\frac{t}{r}\right)_{\max} = \frac{-1.5 + \sqrt{\dfrac{400}{p} - 1.75}}{\dfrac{200}{p} - 2}, \quad . \quad . \quad . \quad (11)$$

where p is per cent error.

For $p = 0$	1	10
$\left(\frac{t}{r}\right)_{\max} = 0$	0.0932	0.260

Note. — For $p = 0$, expression (11) becomes $\frac{\infty}{\infty}$. To evaluate this indefinite form, multiply the numerator and the denominator by p before putting $p = 0$. In the solution of this and of the preceding problem the last term in expression (10) is neglected on account of its relative insignificance; otherwise it would be necessary to solve a complicated cubic equation, and the results would be very little different from those obtained above.

Prob. 9. — Evaluate expression (11) for the value $p = 0$, according to the general rule given in the calculus for indeterminate expressions of the form $\frac{0}{0}$.

Prob. 10. — The required moment of inertia of the rim of a flywheel is 9.25 m^5. The wheel is to run at 75 rev. per min., and its peripheral speed must not exceed 25 m. per sec. (safety requirement). Considerations of space limit the width of the rim to 30 cm. Determine the necessary thickness t of the rim.

Ans. $r = 3.18$ m.; $t = 16.5$ cm.

Hint. — Neglect all the terms but the first in Eq. (10) and calculate t in the first approximation. With the value of t so obtained calculate the expression in the brackets, and again solve for t outside the brackets. This will give the value of t in a second approximation. If necessary, repeat the process once more.

Prob. 11. — What is the moment of inertia of a solid disk, of radius r and width b, about its geometric axis perpendicular to r?

Ans. $\frac{\pi b r^4}{2}$.

Prob. 12. — Referring to the preceding problem, divide the disk into ten concentric parts, each of which contributes the same

amount to the total moment of inertia. Show the concentric circles on a drawing to scale.

Ans. $r_1 = r\sqrt[4]{0.1} = 0.5623\,r$; $r_9 = r\sqrt[4]{0.9} = 0.9740\,r$.

Prob. 13. — Let the concentric parts, considered in Prob. 12, be removed one by one, beginning with the inner one. Plot curves or tabulate values showing: (*a*) the amount of reduction in the moment of inertia in per cent of the moment of inertia of the solid disk; (*b*) amount of material removed, in per cent of the weight of the solid disk; (*c*) the moment of inertia per unit volume. These curves or tables are intended to show the advantage of concentrating as much material as possible near the periphery of a flywheel, where the speed is at its maximum; in this way the largest possible moment of inertia is obtained with the smallest amount of material.

Ans.

Number of cylinders removed from interior.	(*a*) Per cent reduction in the moment of inertia	(*b*) Per cent of material removed.	(*c*) Moment of inertia per unit volume
0	0	0	$0.5\,r^2 \times 1.000$
1	10	31.6	$0.5\,r^2 \times 1.316$
9	90	94.9	$0.5\,r^2 \times 1.949$
10	100	100	$0.5\,r^2 \times 2$

Prob. 14. — A solid disk, of given dimensions and weight, is replaced by another which is n times as thin, and has the same moment of inertia as the first. What is the weight of the new disk, as compared to that of the old one, provided that the same kind of material is used in both?

Ans. The new disk is $\sqrt{n}$ times as light.

Prob. 15. — Formula (6) suggests the following practical expression for the moment of inertia:

$$I = k^2 \cdot v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where v is the total volume of the revolving part, and k is the fictitious radius at which the whole volume must be concentrated in order to give a moment of inertia equal to the actual moment of inertia of the body. Ex. (12) is much used in practice, and k is called the *radius of gyration*. Calculate k for a hollow cylinder of given dimensions.

Solution. — The volume of a hollow cylinder is $\pi b\,(r^2 - r_0^2)$, so that equating (8) and (12) gives

$$k^2 = \frac{(r^4 - r_0^4)}{2\,(r^2 - r_0^2)} = \frac{(r^2 + r_0^2)}{2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Prob. 16. — Prove that the radius of gyration of a hollow cylinder is always greater than the average radius r_m.

Proof. — Eq. (13) can be written in the form

$$k^2 = \frac{1}{2}[(r_m + 0.5\,t)^2 + (r_m - 0.5\,t)^2] = r_m^2 + 0.25\,t^2, \quad . \quad . \quad (14)$$

where t is the thickness of the wall of the cylinder. Since $0.25\,t^2$ is positive, $k > r_m$.

Note. — With values of t small compared with the radius r_m, the radius of gyration k is practically equal to r_m, and the latter is often used in flywheel calculations for the sake of simplicity. This is the more legitimate, as the error is on the safe side, that is to say, the actual moment of inertia is larger than the one calculated. See also Prob. 18.

Prob. 17. — Show that curve (*c*) in Prob. 13 gives values of k^2; supplement the curves or the table by one giving the values of k.

Ans.

No. of cylinders removed,	0	1	9	10
Radius of gyration k,	$0.707\,r$	$0.811\,r$	$0.987\,r$	r

Prob. 18. — Plot a curve showing per cent error between the radius of gyration and the average radius of the rim for rims having, for the ratio of $\frac{t}{r_m}$, values from zero to two. The first limit corresponds to an infinitely thin hollow cylinder, the latter to a solid cylinder. Plot also the per cent error in the moment of inertia when the average radius of the rim is used instead of the radius of gyration.

Solution. — From Eq. (14), $\frac{(k - r_m)}{k} = 1 - \frac{1}{\sqrt{1 + c}}$, where $c = \left(\frac{t}{2\,r_m}\right)^2$. With small values of c, the foregoing formula is inconvenient for calculations. Expanding therefore $(1 + c)^{-\frac{1}{2}}$ according to the binomial theorem, we obtain: $\frac{(k - r_m)}{k} = \frac{2}{c} - \frac{3\,c^2}{8} + \ldots$ Similarly $\frac{(k^2 - r_m^2)}{k^2} = \frac{c}{(1 + c)} = c - c^2 + \ldots$

When	$\frac{t}{r_m} = 0$	$c = 0$	$\frac{(k - r_m)}{k} = 0\%$	$\frac{(k^2 - r_m^2)}{k^2} = 0\%$
	“ = 0.5	“ = 0.0625	“ = 2.98	“ = 5.88
	“ = 1.0	“ = 0.25	“ = 10.6	“ = 20
	“ = 2.0	“ = 1.00	“ = 29.3	“ = 50

Prob. 19. — Write a general expression for the moment of inertia of a rim of variable width (Fig. 16).

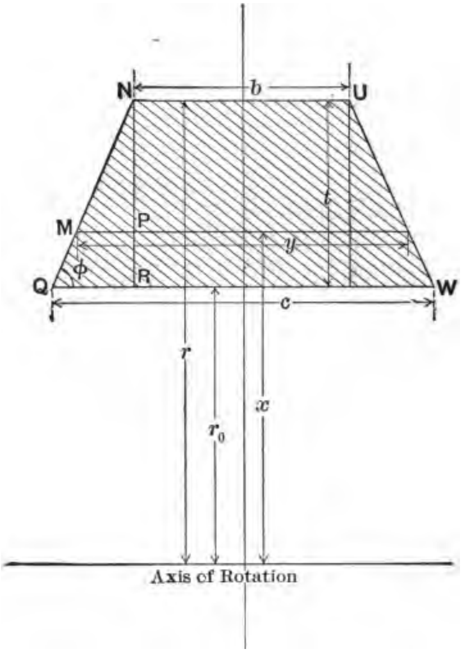

Fig. 16. — A flywheel with the rim of a trapezoidal cross section.

Solution. — Let y be the variable width, parallel to the axis of the shaft; then

$$dv = 2\pi x \cdot y \cdot dx. \quad \ldots\ldots \quad (15)$$

Substituting this into (6) gives

$$I = 2\pi \int_{r_0}^{r} y x^3 \, dx. \quad \ldots\ldots \quad (16)$$

In this expression y must be given as a function of the distance x.

Prob. 20. — Determine the moment of inertia of a rim having a trapezoidal cross section $QNUW$ (Fig. 16), the parallel sides being b and c, the height t.

Solution. — Let $\phi = \angle RQN$ be the angle of inclination of the nonparallel sides; then, from the triangle MNP, $MP = NP \cot\phi$, or $\frac{1}{2}(y - b) = (r - x)\cot\phi$; whence

$$y = (b + 2r\cot\phi) - 2x\cot\phi. \quad . \quad . \quad . \quad . \quad (17)$$

Substituting this value of y into (16) gives, after integration:

$$I = \pi\{(\tfrac{1}{2}b + r\cot\phi)(r^4 - r_0^4) - 0.8\cot\phi\,(r^5 - r_0^5)\},$$

or, reducing to a form more convenient for caculations,

$$I = \pi r^5\left\{\left(\frac{b}{2r} + \cot\phi\right)\left[1 - \left(\frac{r_0}{r}\right)^4\right] - 0.8\cot\phi\left[1 - \left(\frac{r_0}{r}\right)^5\right]\right\}. \quad . \quad (18)$$

When $\left(\frac{r_0}{r}\right)^4$ is small compared with unity, Eq. (18) is simplified to

$$I = \pi r^5\left(\frac{b}{2r} + 0.2\cot\phi\right). \qquad (18a)$$

The value of $\cot\phi$ in these expressions is determined from the triangle QNR:

$$\cot\phi = \cot\angle RQN = \frac{(c - b)}{2t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Prob. 21. — Show that for a rectangular cross section formula (18) becomes identical with (8), and formula (18a) with the answer to Prob. 11.

Prob. 22. — How should formula (18) be modified for numerical calculations when t is small compared with r?

Ans.
$$I = 2\pi t r^3\left\{b\left[1 - 1.5\left(\frac{t}{r}\right) + \left(\frac{t}{r}\right)^2 - \ldots\right]\right.$$
$$\left. + r\cot\phi\left[\left(\frac{t}{r}\right) - 2\left(\frac{t}{r}\right)^2 + \ldots\right]\right\}. \quad . \quad . \quad (20)$$

Hint. — See solution to Prob. 5.

Prob. 23. — Apply formulæ (18) and (20) to the following case: b = 20 cm.; c = 30 cm.; t = 45 cm.; r = 3 m.

Ans. $I = 15$ m.^{5} (nearly).

Prob. 24. — What is the radius of gyration, and the average radius of the rim, in the preceding problem?

Solution. — According to Guldinus' theorem (see Prob. 26), the volume of the rim, $v = S \cdot 2\pi(r - t')$, where S is the area of the cross section of the rim, and t' is the distance of the center of gravity of the cross section from its outside edge NU (Fig. 16). But $S = \frac{(b + c)t}{2}$, and $t' = \frac{t}{3} \cdot \frac{(2c + b)}{(c + b)}$. Substituting the numerical data, we find $v = 1.951$ cu. m. Hence, according to Eq. (12), $k = \sqrt{\frac{15}{1.951}} = 2.773$ m. The mean radius of the rim $r_m = 3.00 - \frac{0.45}{2} = 2.775$ m. Thus, in the case under consideration, it is accurate enough for most practical purposes to calculate the moment of inertia by multiplying the volume of the rim by the square of its average radius, instead of using the more complicated formula (18).

Note. — Expressions for t' used above will be found in various engineering handbooks and pocket books, also in some works on calculus and mechanics, in the chapters on center of gravity of plane figures. If the student prefers to calculate the volume of the rim without the use of Guldinus' theorem, or expression for t', he may do so, as is indicated in the next problem. As a rule, however, it is recommended that the student of engineering should use all possible helps in the form of auxiliary relations, tables, etc., when solving a particular practical problem, and not to shun them simply because he did not have them in his course in calculus or mechanics.

Prob. 25. — Determine the volume v required in the preceding problem, without the use of Guldinus' theorem.

Ans. $v = 2\pi r^3 \left\{ \left(\frac{b}{2r} + \cot\phi \right) \left[1 - \left(\frac{r_0}{r} \right)^2 \right] - \frac{2}{3} \cdot \cot\phi \left[1 - \left(\frac{r_0}{r} \right)^3 \right] \right\}.$

Hint. — Substitute the value of y from Eq. (17) into the general expression (15), and integrate between the limits r_0 and r.

Prob. 26. — Prove that the volume of a body of revolution is equal to the area of its cross section, multiplied by the circumference of the circle passing through the center of gravity of this area. This is the well-known Guldinus' theorem used in Prob. 24 above.

Proof. — According to formula (15), the volume of a body of revolution is

$$v = 2\pi \int xy\,dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

But, according to the definition of the center of gravity,

$$\int y\,dx \cdot x = Sx_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21a)$$

where x_0 is the distance of the center of gravity of the area from the same axis from which distances x are counted; in this particular case, from the axis of rotation. Comparing Eq. (21) and (21a) we get: $v = 2\pi x_0 \cdot S$, which proves the theorem.

Prob. 27. — What is the moment of inertia of a spinning top having a rim of circular cross section of radius a (Fig. 17)? Only an approximate solution is required, assuming the radius of gyration to be equal to the average radius $(r - a)$ of the rim; an exact solution is considered in the next problem.

$$\textit{Ans.} \quad I = 2\pi^2 a^2 r^2 (r - a) \left\{ 1 - 2\left(\frac{a}{r}\right) + \left(\frac{a}{r}\right)^2 \right\}. \quad . \quad . \quad (22)$$

Prob. 28. — What is the exact expression for the moment of inertia required in the preceding problem?

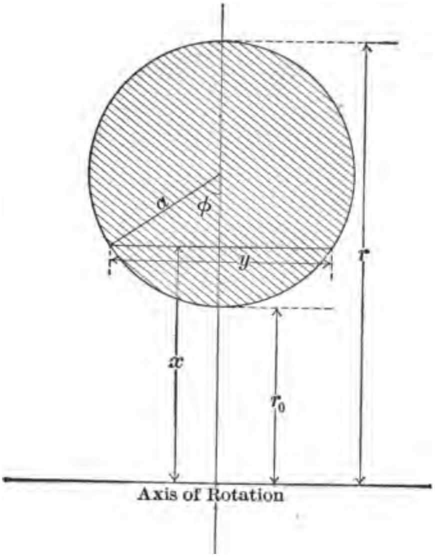

Fig. 17. — A cross section of a flywheel with a circular rim.

Solution. — This is a specific case of integration of expression (16); instead of expressing y as a function of x, it is more convenient in the case of a circle to express both x and y through the central angle ϕ (Fig. (17). We have: $\frac{y}{2} = a \sin \phi$; $x = r - a - a \cos \phi$; $dx = a \sin \phi \, d\phi$. Substituting these values into Eq. (16), this expression involves four integrals whose values are as follows:

$$\int_0^{\pi} \sin^2 \phi \, d\phi = \frac{\pi}{2}; \quad \int_0^{\pi} \sin^2 \phi \cos \phi \, d\phi = 0;$$

$$\int_0^{\pi} \sin^2 \phi \cos^2 \phi \, d\phi = \frac{\pi}{8}; \quad \int_0^{\pi} \sin^2 \phi \cos^3 \phi \, d\phi = 0.$$

So that

$$I = 2\pi^2 ar(r-a)\left\{1 - 2\left(\frac{a}{r}\right) + 1.75\left(\frac{a}{r}\right)^2\right\}. \quad . \quad . \quad (23)$$

It will be seen that for small values of the ratio $\frac{a}{r}$ this expression is practically identical with Eq. (22).

Prob. 29. — Solve the foregoing problem without introducing angle ϕ.

Hint. $y = 2a\sqrt{1 - z^2}$, where $z = \frac{r - a - x}{a}$.

Prob. 30. — At what ratio of $\frac{a}{r}$ does the error in Eq. (22) reach n per cent, as compared with the correct solution (23)?

$$\textit{Ans.} \quad \frac{a}{r} = \frac{\left(\sqrt{\frac{75}{n} - 0.75} - 1\right)}{\left(\frac{75}{n} - 1.75\right)}.$$

For instance, when $n = 1$, $\frac{a}{r} = 0.104$.

Prob. 31. — A disk of variable width is to be designed so that each infinitesimal concentric layer of material should contribute the same amount to the total moment of inertia. What is the shape of the profile of the disk?

Ans. $yx^3 =$ Constant [according to Eq. (16)].

Prob. 32. — Referring to the preceding problem, draw to scale the cross section of a solid steel disk, which must satisfy the following conditions: The disk is to run at a speed of 1500 rev. per min., the peripheral speed not to exceed 80 m. per sec.; the diameter of the shaft is 20 cm.; the total energy of the disk must be 8100 kg-m. (without shaft). Specific gravity of steel is 7.86.

Ans. Diameter of the disk is 1.018 m.; width at the periphery = 2.41 mm.; width at the shaft = 31.8 cm.

Prob. 33. — What is the general expression for the moment of inertia of each arm (or spoke) of a flywheel (Fig. 15)?

Ans. According to Eq. (6)

$$I = \int_{r_1}^{r_0} x^2 \cdot s\,dx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where s is the variable cross section of the arm, and must be given as a function of x.

Prob. 34.— Integrate expression (24) in the first approximation, by assuming the cross section of the arm constant and equal to its mean value s_m.

$$\textit{Ans.} \quad I = \tfrac{1}{3}\, s_m (r_0^3 - r_1^3). \qquad (25)$$

Prob. 35. — Prove that, for a rough approximation, the moment of inertia of the arms can be taken into account by assuming one-third of their weight concentrated in the rim.

Solution. — Expression (25) can be represented in the form

$$I = \tfrac{1}{3}\, s_m \cdot (r_0 - r_1)\,(r_0^2 + r_0 r_1 + r_1^2).$$

But $s_m \cdot (r_0 - r_1)$ is approximately equal to the volume V of the arm, so that

$$I = \frac{1}{3} V r_0^2 \left[1 + \frac{r_1}{r_0} + \left(\frac{r_1}{r_0}\right)^2\right].$$

The two last terms within the brackets are small as compared with unity; on the other hand, r_0 is somewhat smaller than the radius of gyration k of the rim. Assuming that the expression within the brackets makes up for this difference, we get approximately

$$I = \tfrac{1}{3} V k^2. \qquad (26)$$

Comparing this with Eq. (12) proves the proposition and justifies the simple practical formula

$$I_{\text{total}} = (W_{\text{rim}} + \tfrac{1}{3} f W_{\text{arm}})\, r_m^2, \qquad (27)$$

which is much used in the preliminary design of flywheels. In this formula W denotes weight, f is the number of arms, and r_m is the average radius of the rim.

Prob. 36. — Calculate the moment of inertia of an arm more accurately than in Prob. 33; namely, assume that the cross section s varies according to the straight-line law, between its extreme values s_0 and s_1.

Ans. The variable cross section is represented by

$$s = A - Bx, \qquad (28)$$

and Eq. (24) gives

$$I = A \frac{(r_0^3 - r_1^3)}{3} - B \frac{(r_0^4 - r_1^4)}{4}, \qquad (29)$$

where the constants A and B are determined by the conditions

$$\left.\begin{aligned} s_0 &= A - B r_0 \\ s_1 &= A - B r_1 \end{aligned}\right\}. \qquad (30)$$

Prob. 37. — Obtain a still closer value for the moment of inertia of an arm, by considering the arms as straight truncated cones.

Solution. — Let the fictitious vertex of the cone be at a distance R from the axis of the flywheel. Then, for a cross section s at a distance x from the axis, we have

$$\frac{s}{s_0} = \frac{(R-x)^2}{(R-r_0)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

Substituting this value of s into Eq. (24) gives after integration and reduction

$$I = \frac{s_0 r_0^3}{3\left(1-\frac{r_0}{R}\right)^2}\left\{\left[1-\left(\frac{r_1}{r_0}\right)^3\right] - 1.5\left(\frac{r_0}{R}\right)\cdot\left[1-\left(\frac{r_1}{r_0}\right)^4\right]\right.$$
$$\left. + 0.6\left(\frac{r_0}{R}\right)^2\cdot\left[1-\left(\frac{r_1}{r_0}\right)^5\right]\right\}. \quad . \quad . \quad . \qquad (32)$$

In this form the expression is convenient for numerical calculations. The correction terms, of comparatively small magnitude, are directly apparent, and the formula resembles the approximate expressions deduced in Probs. 33 and 34. The distance R is determined from Eq. (31), by applying it to $x = r_1$; we have, then,

$$\frac{s_1}{s_0} = \frac{(R-r_1)^2}{(R-r_0)^2},$$

whence

$$\frac{r_0}{R} = \frac{\left(\sqrt{\frac{s_1}{s_0}} - 1\right)}{\left(\sqrt{\frac{s_1}{s_0}} - \frac{r_1}{r_0}\right)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

When the arms are cylindrical in shape, $s_1 = s_0$; $R = \infty$; $\frac{r_0}{R} = 0$; and Eq. (32) becomes identical with (25), as is to be expected.

Prob. 38. — A flywheel has a diameter of 4.8 m., and the radial thickness of its rim is 38 cm. The wheel is provided with 12 arms of elliptical cross section; the principal dimensions of the cross section at the rim are 4.8 by 8 cm., and at the hub 6 by 10 cm. The arms are 1.77 m. long. Calculate the moment of inertia of one arm, according to the three approximations considered in the previous problems.

Ans. I = 0.0106 m.^{5} according to formula (25);
0.0096 m.^{5} according to formula (29);
0.0095 m.^{5} according to formula (32).

Note. — Dr. F. R. Sharpe has called the author's attention to the following useful formula for calculating the moment of inertia of a rim. Let the distance x between the axis of rotation and a point on the rim be represented as a sum of two distances, $x_0 + x_1$. Here x_0 is the distance between the axis of rotation and a parallel axis passing through the center of gravity of the cross section of the rim; this axis will be referred to below as the axis BB; x_1 is the distance from the axis BB to the point under consideration. Besides, in Eq. (6) we can put $dv = 2\pi x \cdot dA$, where dA is an infinitesimal element of cross section at the point under consideration. Substituting into Eq. (6), we get

$$I = \int 2\pi (x_0 + x_1)^3 \, dA = 2\pi \left\{ x_0^3 A + 3 x_0^2 \int x_1 dA \right.$$
$$\left. + 3 x_0 \int x_1^2 dA + \int x_1^3 dA \right\}.$$

The first of the three integrals is equal to zero, being the static moment of the cross section with respect to the axis BB, passing through the center of gravity. We have, therefore,

$$I = 2\pi \{ x_0^3 A + 3 x_0 K_2 + K_3 \}.$$

Here K_2 is the moment of inertia of the cross section with respect to the axis BB. For usual forms of cross section, expressions for K_2 are given in engineering pocket books. K_3, for lack of a better name, can be called the *cubic moment* of the cross section, with respect to the axis BB. If the cross section is symmetrical with respect to the axis BB, $K_3 = 0$, because to each positive value of $x_1^3 dA$ corresponds an identical negative value. Thus, for symmetrical cross sections, we have simply

$$I = 2\pi x_0 \{ x_0^2 A + 3 K_2 \}.$$

For unsymmetrical cross sections the value of K_3 has to be calculated, so that the method does not possess any advantage over that given in the text above. However, with moderate unsymmetry in the shape, the value of K_3 is very small as compared with the two other terms in the expression for I. It can, therefore, be neglected, or calculated under simplifying assumptions.

LIST OF REFERENCE WORKS ON MATHEMATICS.

(See also the books recommended in the appendix.)

TRIGONOMETRY

Author.	Publisher.
Ashton & Marsh	Chas. Scribner's Sons.
Moritz	John Wiley & Sons.
Jones	Geo. W. Jones (Ithaca).
Hobson	Cambridge University Press.

ANALYTIC GEOMETRY

Author.	Publisher.
Tanner & Allen (revised edition)	Amer. Bk. Co.
Smith & Gale (larger edition)	Ginn & Co.
Fine & Thompson	The Macmillan Co.
C. Smith	The Macmillan Co.
Salmon	Longmans, Green & Co.

DIFFERENTIAL AND INTEGRAL CALCULUS

(*Both in one volume.*)

Author.	Publisher.
Snyder & Hutchinson	Amer. Bk. Co.
Granville	Ginn & Co.
Murray	Longmans, Green & Co.
Osgood	The Macmillan Co.
Echols	Henry Holt & Co.
Lamb	University Press.
Greenhill	The Macmillan Co.

DIFFERENTIAL CALCULUS

Author.	Publisher.
McMahon & Snyder	Amer. Bk. Co.
Byerly	Ginn & Co.
Edwards	The Macmillan Co.
Williamson	Longmans, Green & Co.

INTEGRAL CALCULUS

Murray	Amer. Bk. Co.
Byerly	Ginn & Co.
Edwards	The Macmillan Co.
Williamson	Longmans, Green & Co.
Todhunter	The Macmillan Co.

HYPERBOLIC FUNCTIONS

McMahon	John Wiley & Sons.
Mathematical Tables	Smithsonian Inst.

TABLES OF INTEGRALS

B. O. Peirce	Ginn & Co.

MISCELLANEOUS

Merriman-Woodward. Mathematical Monographs.
John Wiley & Sons.

No. 1. History of Modern Mathematics, by David Eugene Smith.
No. 2. Synthetic Projective Geometry, by George Bruce Halsted.
No. 3. Determinants, by Laenas Gifford Weld.
No. 4. Hyperbolic Functions, by James McMahon.
No. 5. Harmonic Functions, by William E. Byerly.
No. 6. Grassman's Space Analysis, by Edward W. Hyde.
No. 7. Probability and Theory of Errors, by Robert S. Woodward.
No. 8. Vector Analysis and Quaternions, by Alexander Macfarlane.
No. 9. Differential Equations, by William Woolsey Johnson.
No. 10. Solution of Equations, by Mansfield Merriman.
No. 11. Functions of a Complex Variable, by Thomas S. Fiske.

LIST OF REFERENCE BOOKS ON MACHINE DESIGN.

BACH	Julius Springer.
BENJAMIN	Henry Holt & Co.
JONES	John Wiley & Sons.
KIMBALL & BARR	John Wiley & Sons.
SMITH & MARX	John Wiley & Sons.
UNWIN	Longmans, Green & Co.

APPENDIX.

WHAT A SENIOR IN ENGINEERING OUGHT TO KNOW ABOUT MATHEMATICS *

By V. Karapetoff.

1. *Fluency in ordinary mathematical transformations and calculations.* College examination problems in elementary mathematics are recommended for practice (College Entrance Examination Board, *Examination Questions in Mathematics.* Ginn & Company, 25 cts.).

2. *Algebra.* The solution of simultaneous and quadratic equations. Go back to your textbook and solve a number of problems on these topics. Theory and practical use of common and natural logarithms. Logarithms of quantities with negative and fractional exponents, and of quantities less than unity. A familiarity with complex quantities will help greatly in the understanding of the theory of alternating currents.

3. *Trigonometry.* Definition of sine, cosine, tangent, cotangent; the fundamental relations between these quantities, and the limits within which they vary. Functions of the sum and of the difference of two angles. The simplest cases of the solution of triangles. Solve a number of examples on these topics from your old textbook on trigonometry.

4. *Differential Calculus.* A clear understanding of the process of differentiation, in the sense of finding the rate of change of a function with respect to the independent variable. Become absolutely sure in the differentiation of simple algebraic expressions, and of those involving sines, cosines, and logarithms. Maxima and minima of functions of one variable.

5. *Integral Calculus.* A clear understanding of the operation of integration, in the sense of finding the area of a curve, and also as anti-derivative. Make clear to yourself the meaning of the limits of integration, and of the constant of integration. Acquire fluency in the integration of simple algebraic expressions, and of those containing sines and cosines.

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* Reprinted from the Proceedings of the Society for the Promotion of Engineering Education, Vol. 18 (1910), p. 271. Much valuable information concerning the scope and the methods of teaching mathematics to engineering students will be found in the various volumes of these proceedings.

6. *Application of mathematics to mechanics and physics.* Go over your textbooks of physics and mechanics, and select topics which involve a considerable use of mathematics. Solve the most important problems in these topics, paying the principal attention to the mathematical methods employed, rather than to the physical side of the questions. This will be a valuable preparation for the application of mathematics to engineering.

GENERAL REMARKS

1. Do not try to cover too much in the above outlined program. Select a topic in which you are particularly interested, or in which you feel particularly deficient, and go over it carefully, before beginning the next topic. One subject in mathematics studied thoroughly will give you more insight into the general mathematical method, than a large field covered in a superficial way and only half understood. Many troubles in the understanding of the calculus have their origin in insufficient preparation in elementary mathematics. Therefore, it is advisable to review the principal parts of algebra and trigonometry before reviewing the calculus.

2. Having selected a topic, begin its study by solving a considerable number of problems, so as to acquire a fluency in applications. Refer back to theory only in so far as you find difficulties in the solution of the problems, or in understanding the reasons for certain relations.

3. A convenient reference book for all the above-mentioned topics in elementary mathematics and in calculus is the Mathematical Handbook by *E. P. Seaver* (McGraw-Hill, $2.50). No proofs are given in this book, but only the principal formulæ and rules, arranged in a way convenient for ready reference. A somewhat larger book is that by *J. Claudel*, — Handbook of Mathematics (McGraw-Hill, $3.50).

4. Those particularly interested in application of mathematics to engineering will find a large number of engineering problems with solutions in the following works:

F. M. Saxelby. A Course in Practical Mathematics (Longmans, $2.25). See in particular the examination papers, beginning on p. 396. This book is particularly recommended for home study.

F. Castle. A Manual of Practical Mathematics (Macmillan, $1.50).

R. G. Blaine. The Calculus and its Applications (Van Nostrand, $1.50).

John Perry. Calculus for Engineers (Longmans, $2.50).

Chas. P. Steinmetz. Engineering Mathematics (McGraw-Hill, $3.00).

5. Every year some seniors have difficulty in grasping the theoretical subjects in engineering, simply because of insufficient preparation

in mathematics. They fail in spite of diligent work, because there is no time during the college year to devote to the review of mathematics. It is therefore urgently recommended that those who feel deficient in mathematics go over the subject during the summer before the senior year. Not only will poor preparation in mathematics be considered no excuse for unsatisfactory work in engineering subjects, but, on the contrary, it may seriously interfere with graduation.

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